From a Model for Spin Glasses to the Phenomenology of Glasses

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For half a century theorists have puzzled over three characteristic behaviors of glasses near the glass transition: the Kauzmann paradox (that the configurational entropy extrapolates to zero at a temperature T_K somewhat below the glass transition Tg); the Vogel-Fulcher law for the structural relaxation time deduced for example from the measured viscosity, $\tau = \tau_0 \exp -A/(T - T_0)$, with $T_0 \approx T_K$; and the Adams-Gibbs stretched stretched-exponential law from the bulk measurements of relaxation , $C(t) \propto \exp -(t/\tau)^\beta$, with $\beta < 1$. This paper shows that all three can be, if not derived on the back of an envelope, at least calculated on an almost trivial model with a simple decimation RNG. The only catch is that the trivial model, the 1D Ising spin glass in a magnetic field, can only be related to real structural glasses by the most tenuous of arguments involving a series of enormous leaps in the dark; not that I fail to believe that the connection is there, but that it seems a very long way around.

The story perhaps starts with the observation by Gotze that behavior like that of glass could be described by a set of kinetic equations for density fluctuations containing a non-linear coupling among the modes truncated in the simplest possible way. There seemed little motivation for this procedure and even the nature of the modes seemed ambiguous, but it seemed to work. The second development was the idea, apparently originating with Kirkpatrick and Wolynes, and soon picked up by Parisi and associates, that structural glass, which has no preexisting disorder, could be related to a spin glass model in which there are random interactions among more than two spins. Such spin glass models have a mean field theory valid for long range interactions, using the replica trick, which is closely related to the mode-coupling equations. Both sets of equations predict an underlying first-order transition at which correlation length becomes infinite, identified with the Kauzmann temperature, but which is never reached because relaxation slows down above it. The argument for why one can describe the true glass

with no built-in disorder by a spin glass with quenched disorder is never really made explicit but involves the statement that the replica equations are in the same universality class as the mode-coupling equations, which at least have no quenched disorder.

The newer developments by Moore and co-workers seem to consist in showing that in finite dimensionality, with no infinite-range interactions, no such transition exists, and the correlation length remains finite at all temperatures. Under these circumstances one may rely on the droplet ideas of Fisher and Huse and do scaling accordingly. When this is done one finds that the actual physical glass is so far from the putative phase transition that even the droplet model is unnecessary. In this regime the nonlinear terms lead to a random time-reversal breaking magnetic field, which of course is no different, in a spin glass, from a T-dependent constant h-field. Since one is far above any T_c, even the dimensionality is irrelevant and one might as well use 1D. Thus the 1D Ising spin glassand simple decimation gives us all the laws of the glass transition. A less exact simulation on a 3D spin glass gives similar results. An additional bonus is that the T-dependence of the h-field is related to Angells fragility parameter $T_K - T_0$.

Is the mysterious nature of glass a solved problem? In principle, these developmentswhich are being pursued by a number of active groups in addition to Moorestell us a lot about this problem. They show that even clusters with a perfectly finite range of correlation can cooperatively slow things down to a virtual halt, and if they do the characteristic glass behavior will follow, even though there is no real underlying phase transition except in a fictive high-dimensional model. But there remans some work to do in deriving the models directly from the molecular structure with fewer gestures.