Semiclassical dynamics and long time asymptotics of the central spin problem

Authors: Gang Chen, Doron L. Bergman, and Leon Balents
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Recommended with a Commentary by Leonid Glazman, Minnesota

The idea of quantum computing and ongoing search for a viable implementation of a quantum bit brought dynamics of individual spins in solids into the spotlight. A single electron in a quantum dot provides an interesting example of an isolated spin. The trapped electron can be prepared in a desired spin state. At a later time, its state can be read out by means of electron transport (see Ref. [14] in the preprint of Gang Chen *et al*).

A quantum dot may appear as a sterile environment allowing a coherent evolution of electron spin. There is one nuisance, however. The material of choice for hosting a quantum dot is GaAs, in which the density of nuclear spins is essentially equal to the atomic density. The electron spin in a quantum dot is exposed to the hyperfine interaction with the nuclei. If all the nuclear spins were polarized, the effective field acting on electron spin would be around 3T. In reality, nuclear spins in the "virgin" state are fully randomized. The number of these spins N in the volume of a quantum dot is about $N \sim 10^6$, so electron spin experiences an effective field $B_{\text{eff}} \sim 30$ Oe.

Experiments with a quantum dot allow one to cycle electrons through it at certain pace, thus sampling different representatives of the ensemble of nuclear spin configurations. Electron spin evolution can be measured for each initial state of the nuclear spins. Upon proper averaging over the cycles, one may find the electron spin correlation function, $F(t) = \langle \vec{S}(t) \cdot \vec{S}(0) \rangle / S(S+1)$. Here $\langle \dots \rangle$ means averaging over the random distribution of the nuclear spins $\vec{I}_i$ which interact with the electron spin. If one replaces the effect of nuclear spins by a *static* field B_{eff} and then average over its directions, then $F(t)$ would display a decay from $F(0) = 1$ to a *finite* value $F(t \to \infty) = 1/3$. The time scale for that evolution is $\propto B_{\text{eff}}^{-1}$. The crude estimate yielding a finite value of $F(t \to \infty)$ completely neglects the back-action of the electron spin on the nuclei. In fact, the nuclear spins also precess due to their interaction with the electron spin (we neglect here the dipole-dipole interactions between the nuclear spins). Would that interaction drive $F(t \to \infty)$ to zero?

This question did attract a considerable interest lately not only in relation with experiments, but also because of the beauty of the model describing the corresponding spin system. In the so-called "central spin" problem, a single electron spin S interacts with a bunch of N nuclear spins I_i,

$$H = \sum_i a_i \vec{I}_i \cdot \vec{S},$$

and the exchange constants a_i are arbitrary. In reality, these constants vary with the amplitude of the electron wave function confined to a quantum dot, $a_i \propto |\psi(\vec{r}_i)|^2$. This model belongs to a broader class of integrable systems ("Gaudin magnets") discovered by M. Gaudin [J. Phys. (Paris) **37** 1087 (1976)]. So far, there was little success in exploiting the rich mathematical structure of the model in order to extract information about correlation function $F(t)$.

Because of the vastly different scales of effective fields exerted on the electron spin ($\sim a\sqrt{N}$) and on a typical nuclear spin ($\sim a$), one may argue in favor of the described above "$B_{\rm eff}$–approximation". This mismatch of the effective fields prohibits a direct spin-flip process involving electron and a single nuclear spin. Perhaps this heuristic argument has certain value, as there is no evidence for an exponential decay of function $F(t)$ down to zero. Nevertheless, there are more complex processes involving many nuclear spins which may affect the dynamics of the electron spin and eventually drive $F(t)$ to zero at $t \to \infty$. The preprint of Chen *et al* presents a compelling argument in favor of a very slow decay, $F(t) \propto |\ln t|^{-\alpha}$. This kind of behavior was observed earlier in numerical simulations (Ref. [13] in the preprint). Chen *et al* associate the slow dynamics of the electron spin with its interaction with the most remote nuclear spins, sitting in the "tail" of the electron wave function ψ. The exponent α is not universal and depends on the asymptotic behavior of ψ at large r (the periphery of the dot). For a two-dimensional dot, $\alpha = 2$ in the case of exponentially decaying wave function.