Topological nature of polarization and charge pumping in ferroelectrics.

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Recommended and a commentary by David Vanderbilt, Rutgers University.

Since the pioneering work of Thouless on the quantization of adiabatic charge transport in insulators in the mid-1980s, the importance oftopological considerations for understanding charge pumping in systems such as sliding charge-density waves and quantum-Hall insulators has been widely appreciated. The same insights essentially underlie the development of the "modern theory of polarization" in the mid-1990s, when it was understood that the electric polarization of an insulator is properly related to a sum over occupied bands of a Berry phase associated with the transport of the Bloch wavefunction on a circuit around the Brillouin zone.

The preprint by Onoda et al. presents a method for an elegant topological analysis of just how the charge pumping can occur, at least in simple cases of conventional insulators undergoing an adiabatic cyclic evolution with respect to some parameters in the periodic Hamiltonian. By a clever mapping, the authors introduce vector fields in the parameter space, and in particular, a "flux density" which vanishes except on strings for which the flux is quantized to integer values. These flux lines are essentially the locus of points in parameter space at which an accidental gap closure occurs at some point in the Brillouin zone. Polarization changes are related to these flux lines in a manner analogous to the way magnetic fields arise from line currents. In particular, quantum charge transport occurs only for cyclic loops in parameter space that enclose such flux lines, and the integer charge pumped is just related to the linking number of the loop about the flux lines. The authors investigate a variety of toy models of insulators in which these flux lines can be seen to circulate, or to meet and branch, in various entertaining, ways depending on the choice of model.

Most of the formal discussion and investigation of models is restricted to the simplest interesting class of problems, namely, the case of non-interacting 1D insulators whose Hamiltonian depends on a family of three parameters. However, I imagine the work will stimulate extensions to more complex cases.

And in any case, I recommend it as an elegant use of topological methods to elucidate the phenomenon of adiabatic charge transport in insulators in terms of our modern understanding of the role of Berry phases in the theory of electronic bandstructure.