Stalactite growth as a free-boundary problem: A geometric law and its platonic ideal.

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Recommended with a Commentary by Mehran Kardar, MIT, and Wim van Saarloos, Institute-Lorentz, Leiden University

Spleothem is the technical term for cave formations such as stalactites and stalagmites, informally known as 'pretties' to cave enthusiasts. These structures form as a result of precipitation of calcium carbonate from the water seeping into the caverns - the oversaturation of the water is due to the outgasing of CO_2 from the water as a result of the lower partial pressure of CO_2 in the cave. The question asked in the paper is what determines their overall shape. Somewhat surprisingly, the authors come to the conclusion that stalactites (on average) have an ideal shape, which they derive mathematically and confirm by observations.

Their first observation is that to maintain a uniform flux of water sliding down the stalactite, the thickness *h* of the fluid film must vary. Simple dimensional analysis using the Stokes equation then yields $h \sim (R \sin\theta)^{-1/3}$, where *R* is the local radius and θ is the angle with respect to vertical. Thus the film gets thicker at the bottom of the stalactite.

The authors next appeal to considerations of chemistry to derive the rate of calcium deposition, and conclude that it is proportional to the film thickness *h*. Given the long times involved, this is not unexpected. The fact that the authors conclude that diffusion of CO_2 though the cave atmosphere is not important, sets this system apart from diffusion controlled free boundary growth problems. These two observations then determine the local growth velocity of the stalactite. This growth law converges to a uniformly translating solution, which is the ideal shape. (Actual growth rates of the tips are of the order of 1cm per century.)

Up to an overall scale factor, the ideal shape is a slender form with a radius that grows as the 4/3 power of the height close to the tip, and logarithmically further up. The authors then go on to photograph and measure a variety of stalactites in Kartchner Caverns (Benson, AZ). They find very good agreement between the actual shapes and the predicted mathematical form.

At least at the level of numerical simulations, the present local model in which convective transport along the stalactite is ignored, does not exhibit a ripple instability seen in practice, but that it may be a starting point forinvestigating such phenomena. Moreover, much of the same physics (a thin fluid layer, diffusion of heat) enters the formation of icecles.