Entropy and Temperature of a Static Granular Assembly

Authors: Silke Henkes, Corey S. OHern and Bulbul Chakraborty
arXiv:cond-mat/0701489v1

Recommended and with a commentary by Sriram Ramaswamy, IISc, Bangalore

Equilibrium statistical mechanics is based on the notion that for a sufficiently complicated system the total energy E is the only smooth integral of the motion. Together with the postulate of equal *a priori* probabilities this allows us to construct, for a system of N particles in a volume V, the entropy $S = \log \Omega(N, V, E)$ as the logarithm of the number of accessible microstates, providing a microscopic basis for thermodynamics. In granular matter, the particles have a macroscopic number of internal degrees of freedom. Interparticle collisions dissipate energy, and this energy is never fed back into the translational or rotational motion of the particles. Left to itself, an initially shaken collection of granular particles will settle down into one of a large number of static, mechanically stable configurations. or "blocked states". Total energy should thus play no role in determining the mechanics and statistics of gently disturbed granular matter, which should instead be governed by how the system explores the space of blocked configurations. Understanding the statistics of these configurations would greatly expand our understand of jamming [Liu and Nagel, Nature **396** (1998) 21]. In particular, is their distribution intrinsic, or does it depend on the preparation history? Henkes *et al.* present a conjecture that tells us how to count and organise blocked states for isotropic collections of particles interacting only by contact. .

In their study of loops of forces around voids in granular packings, Ball and Blumenfeld (ref [4] of the preprint) use the constraints of force and torque balance to show that the macroscopic stress tensor associated with a region is determined entirely by a sum over its boundary. Noting that the total stress is therefore a conserved quantity in any dynamics consisting of local rearrangements within the space of blocked configurations, Henkes *et al.* use its trace Γ as a natural scalar invariant in place of the energy, and postulate that the statistical mechanics of such systems maximises the entropy $S = \ln \Omega(\Gamma, N, V)$. With this microcanonical assumption in hand, Henkes *et al.*

define the analogue of temperature using Γ in place of the energy and make several predictions, the simplest that the analogue of thermal equilibrium is that "temperature" defined as $(\partial S/\partial\Gamma)^{-1}$ must be the same everywhere, and hence that the local value of Γ in regions within a large system should be canonically distributed.

The authors test their predictions against numerically generated packings of deformable circles, and find remarkably good agreement. It is particularly interesting that the measured dependence of the effective temperature on the "energy" Γ turns out to be consistent with an entropy proportional to $\ln\Omega$, lending credence to the idea that all microstates with a given Γ are equally likely. Lastly, this work is compelling evidence in favour of the early (and at the time quite mysterious) conjectures by Edwards (ref [7] of the preprint), on a microcanonical ensemble for rigid grains, based on total volume and total external force. .