A Class of P,T-Invariant Topological Phases of Interacting Electrons.
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The quantum theory of solids rests on the Fermi liquid paradigm, which in effect states that ground states of interacting electrons can be put into a one-to-one correspondence with a reference non-interacting state. This framework not only gives an adequate description of simple metals, insulators and semiconductors, but also can account naturally for electronic phases with broken symmetries arising from pairing electrons and or holes, such as BCS superconductivity, charge/spin density waves and itinerant magnetism. With the discovery of the fractional quantum Hall effect (FQHE) in the early 1980's, it became clear that the Fermi liquid paradigm sometimes fails completely, and that a new theoretical framework is required to describe the resulting non-Fermi liquid ground state. Indeed, the FQHE phases possess a novel form of internal structure - a ``topological order" - which is qualitatively distinct from conventional ordered states (a Ferro magnet, say) which involve a spontaneous symmetry breaking. As with other topologically ordered phases, the 2d FQHE ground states are non-degenerate when placed on surfaces of non-trivial topology, such as a torus. These degeneracies are intimately linked to the emergence of quasiparticle excitations with fractional quantum numbers. Sometimes the quasiparticles exhibit non-abelian braiding statistics, and multi-particle states then form an error resistant Hilbert space, which might be useful for quantum computation. Generally, such topological quantum states involve the emergence of a gauge theory description - a Chern-Simons gauge theory for the FQHE.

In the recent preprint Michael Freedman, Chetan Nayak, Kirill Shtengel, Kevin Walker and Zhenghan Wang, construct a class of topological phases of 2d electrons, which in contrast to the FQHE states can occur in time reversal invariant systems (zero magnetic field). These states also afford a gauge theory description, often ``doubled" versions of Chern-Simons theory. A central theme in this highly pedagogical preprint, is that these topological quantum field theories can be profitably recast in the language of curves living on 2d surfaces. The full topological content is encapsulated in several simple ``surgery" and ``fusion" rules, dictating how the various curves can be joined and raided. This appealingly accessible description of 2d topological quantum field theories is apparently familiar to topologists, but is not widely appreciated within the condensed matter theory community.

This preprint offers a clear and simple dictionary between the formulation in terms of curves and the more familiar gauge theory formulations. Starting with the simplest abelian Chern-Simons theories and then moving to the discrete Z2 gauge theory, Freedman et. al. shows that the Lagrangian or Hamiltonian formulation of such gauge theories can be recast in terms of gauge invariant Wilson-loop operators. When acting on the ground state, these operators create loop-like excitations, which serve as the building block for the eventual topological description in terms of curves on surfaces. Having laid this foundation, Freedman et. al. next analyze a class of topological quantum field theories which have excitations with non-abelian braiding statistics, and illustrate how these highly non-trivial excitations can again be understood pictorially in terms of curves on surfaces with their particular surgery and fusion rules.

This preprint is strongly recommended for any condensed matter theorist with an interest in exotic non-Fermi liquid phases of 2d electrons, and also for those with an interest in applying topological field theories to quantum computing. It is self-contained and accessible, with a lengthy listing of helpful references. Condensed matter theorists expert in the FQHE will find within an entirely new framework to understand and access the topological structure underlying such phases. This paper could well serve as a departure point for numerous further theoretical developments.