Supersolid order on the triangular lattice.

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http://ArXiv.org/cond-mat/0505298

Phys. Rev. Lett. **95**, 127205 (2005);

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http://ArXiv.org/cond-mat/0505257

Phys. Rev. Lett. **95**, 127206 (2005);

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Recommended with a commentary by Subir Sachdev, Harvard University

The last year has seen a flurry of activity on the supersolid state. This state was hypothesized in 1969-70 by Andreev, Lifshitz, and Chester as a possible phase of ^{4}He, by considering the quantum mechanics of vacancies or interstitial defects in solid ^{4}He. These defects are expected to become mobile via quantum tunneling at low temperatures, and so behave like a dilute gas of bosons. It was argued that these bosons can undergo Bose condensation without disrupting the crystalline order of the host crystal, thus leading to the 'supersolid': a state of matter with two distinct broken symmetries, the broken translational symmetry associated with the crystalline order, and the broken U(1) symmetry of the superfluid state. Such a state can support both superflow and the shear waves of the crystal.

The recent interest has partly been stimulated by experiments at Penn State (E. Kim and M. Chan, Nature (London) **427**, 225 (2004); Science **305**, 1941 (2004)) on ^{4}He. The status of these experiments have been addressed by other comments in this journal club, and I will not discuss it here. Numerical studies of realistic ^{4}He models strongly indicate that the pure ^{4}He does not support a homogeneous supersolid phase.

A distinct motivation for the study of supersolids comes from the cuprates and other correlated electron compounds. Here we consider a slightly different 'lattice supersolid' phase. The host crystalline lattice can be assumed to be rigid, so the symmetries of continuous translations and rotations of free space are broken in all phases. Nevertheless, there is an unbroken discrete space group symmetry of the lattice, and we can ask if the electronic state spontaneously breaks this space group symmetry. A superconducting state with a spontaneously broken space group symmetry is then a 'lattice supersolid'. There is evidence for the breaking of square lattice symmetry in a number of doped cuprates, in the form of phases variously identified as 'stripe/checkerboard/charge-density-wave'. Some of these phases also exhibit a non-zero T_c for superconductivity, although it is not completely clear if the 'stripe' order and superconductivity are properties of the same phase, or manifestation of different phase-separated regions of the sample. Nevertheless, the observations do raise the possibility that a lattice supersolid exists in a generalized cuprate phase diagram.

A notable aspect of the broken lattice symmetry has emerged in recent studies of a closely-related spin ladder compound (A. Rusydi, P. Abbamonte, M. Berciu, S. Smadici, H. Eisaki, Y. Fujimaki, S. Uchida, M. Ruebhausen, and G. A. Sawatzky, cond-mat/0604101): each unit cell of an insulating state with broken lattice symmetry contains exactly one pair of holes, or an integer number of Cooper pairs. Theoretically, this suggests that a model of interacting lattice bosons, each boson representing a Cooper pair, may be a reasonable starting point for investigating the interplay between broken lattice symmetry and superfluidity.

There have been a number of theoretical studies of the ground state phase diagram of hard-core lattice bosons. At half-filling on the square lattice, with strong nearest-neighbor repulsive interactions between the bosons, the ground state is a two-fold degenerate checkerboard insulator. Moving away from half-filling, we then introduce vacancies or interstitials in the cherckerboard state. The defect-condensation method discussed above applies here, and could lead to a supersolid state. However, a variety of numerical studies have shown that the situation is rather delicate, and quite prone to phase separation into a half-filled insulator and a superfluid. Longer-range interactions or hopping terms are certainly needed to stabilize a possible, homogeneous supersolid phase.

The papers highlighted in this note introduce a new mechanism for the stabilization of a lattice supersolid. They consider hard-core bosons at half-filling on the triangular lattice. This introduces the additional ingredient of frustration, and associated degeneracies in the insulating state. Indeed, in the limit of zero boson hopping, the ground state has a macroscopic degeneracy for the model with only nearest neighbor repulsion: there are an infinite number of arranging bosons on the triangular lattice so that half the sites are occupied and only one-third of the bonds have bosons on both ends. Boson-boson correlations in this ensemble can be computed exactly, and it is known that correlations at certain wavevectors decay with a power-law.

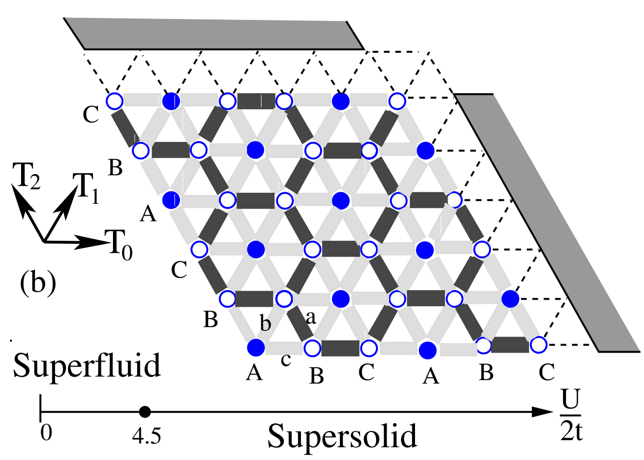
Now examine the influence of introducing a small boson-tunneling amplitude into this ensemble of degenerate insulating states. The first three papers listed above all demonstrate convincingly that two important phenomena occur:

(i) For infinitesimal tunneling, the ground state is simply the equal superposition state of all the degenerate insulators. Strong numerical evidence is presented that this state has off-diagonal long-range order, with a finite superfluid stiffness.

(ii) For small, but finite, tunneling, the dynamics within the low energy manifold can be mapped onto a quantum dimer model on the dual hexagonal lattice. This dimer model was shown some time ago to exhibit 'valence-bond-solid' (VBS) order. For the bosons on the triangular lattice, this VBS order translates into true long-range order at the wavevector at which there was power-law order for zero tunneling. This emergence of a broken lattice symmetry can be considered to be an 'order-by-disorder' effect induced by quantum fluctuations.

Taken together, these effects convincingly demonstrate that the half-filled boson model on the triangular lattice is a supersolid.

A sketch of the structure of the supersolid phase is shown in the figure. In a loose sense,



we can view this supersolid as exhibiting phase separation into solid and superfluid, but on a microscopic scale. As shown in the figure, one of the triangular sublattices has one boson per site (this accounts for 1/3 filling), forming a static insulator which forms the 'solid', and ensures the breaking of lattice symmetry. The remaining density of 1/6 filling bosons forms a superfluid on which resides on the honeycomb lattice of bonds indicated by the thick lines.

In the last paper cited above, Melko *et al.* have extended the study of the hard-core boson model on the triangular lattice to include strong 2nd neighbor repulsion. They find a new supersolid phase, in which the solid order is *striped.* Again, the supersolid phase has a 'microscopic phase separation' interpretation, with an insulator of filled rows (separated by 3 lattice spacings) interspersed by mobile atoms forming the superfluid. However, there are strong indications from the numerics that this interpretation is too simplistic: the anisotropy in the superfluid stiffness is much smaller than that in the solid order.