**Entropy Maximization in the Force Network Ensemble for Granular Solids**

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**Recommended with Commentary by R.P. Behringer, Duke University**

An ongoing debate in the soft condensed matter community concerns the distribution of contact forces, $P(f)$, for jammed disordered solids. A jammed system has a non-zero yield stress, or alternatively, non-zero elastic coefficients. In pioneering work, Liu et al.[1] combined theory and experiments to yield the first results for $P(f)$, which was an exponential: $P(f) \propto \exp(-f)$. The experiments have become a classic in the field, and are referred to as carbon-paper experiments for the use of carbon paper at the boundaries to measure the particle-wall forces.

Since then, there have been a number of new models and experiments that have sought to understand the nature of $P(f)$ and its origins from a fundamental statistical approach. In fact, the theoretical ground work for the statistical basis was put forward by Edwards and Oakeshott [2], in the form of the Edwards entropy. The original Edwards ensemble considered packings of $N$ infinitely rigid particles in a fixed volume, $V$. The Edwards entropy is then given as the logarithm of the number of jammed states that are consistent with a given $N$ and $V$, assuming an equal probability of occurrence for any admissible state. In this case, the volume is the key control parameter, and force is irrelevant.

This picture was later generalized to particles of finite elastic properties, where forces are important[3]. The whole was taken up by other groups, who have considered ensembles of force-balanced states, starting with the work of Snoeijer et al.[4] and including work by Tighe et al.[5] and by Henkes et al.[6], among others. In a recent study, van Eerd et al. [7] have obtained numerical results indicating a nearly gaussian distribution in 2D, and a modified gaussian, $P(f) \propto \exp(-f^{\alpha})$ with exponent $\alpha \approx 1.7$ in 3D.

The key contribution of the present work by Tighe et al. is the observation that for isotropic two-dimensional packings of frictionless particles, at least, there are two conserved quantities, the pressure, and a conserved tiling area, based on an observation due to Maxwell. Armed with these two conservation principles, Tighe et al. argue that the generic form for $P(f)$ in 2D should be a gaussian in $f$, with an exponential as an extreme limit in certain cases, such as infinite friction. Their argument depends on an assumed lack of correlations, and on knowing a density of states, which they estimate in the absence of correlations. They then support their assertion through various simulations.

So, how does the Tighe et al. result fit with experiments, and in particular, the experiments of Liu et al, which showed a clear exponential for $P(f)$? There are perhaps two key points to consider. First, Snoeijer et al.[8] have argued that there are correlations induced by walls, and that the bulk $P(f)$ can only be obtained from interior grains. This may give some insight into why the carbon-paper experiments yielded such a convincing exponential. Second, the stress state, and in particular, its anisotropy is likely very important. In particular, experiments on 2D granular systems by Majmudar et al.[9], and calculations in the force ensemble by Snoeijer et al.[4] and by Tighe et al.[5] have shown that $P(f)$ has a gaussian-like tail for isotropic compression, but a more nearly exponential tail for pure shear. For 3D granular systems, there are no experimental determinations of the forces on interior grains, to my knowledge. But there are two elegant experiments by Brujić et al. [10] on centrifuged emulsions and by Zhou et al.[11] on dense liquid drop suspensions. The former experiments suggest a more exponential $P(f)$, but the latter are much more gaussian in character.

The weight of the evidence, particularly with this recent work by Tighe et al., is pointing towards gaussian tails for isotropic stress states of zero or low-friction particles in the 2D case. Perhaps the jury is still out, but the evidence is mounting that the tails are either gaussians in 2D or modified gaussians with exponents bigger than 1 but perhaps less than 2 in 3D.

In fact, I would argue that the big issue is not gaussian tail vs. exponential tail. Rather, the bigger issue is to what extent are the generalized Edwards pictures relevant to physical granular systems? Force-ensembles are based on conservation principles, so they may be our best hope for a physically based statistical description for granular and related disordered systems. Hence, the fuss about the shape of $P(f)$ has real substance.

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