Simulating the Hubbard Model by Cold Atoms.

1. "Many-Body Physics with Ultracold Gases",
Bloch, J. Dalibard and W. Zwerger,
arXiv:0704.3011; to be published in Rev. Mod. Phys.

2. "A Mott insulator of fermionic atoms in an optical lattice",
R. Jördens, N. Strohmaier, K. Günter, H. Moritz, T. Esslinger,
arXiv:0804.4009v1, Nature **455** 204 (2008).

3."Metallic and Insulating Phases of Repulsively Interacting Fermions in a 3D Optical Lattice", U. Schneider, L. Hackermüller, S. Will, Th. Best, and I. Bloch, T. A. Costi, R. W. Helmes, D. Rasch, and A. Rosch,
arXiv:0809.1464v1.

4. "Trapping and cooling fermionic atoms into the Mott and Neel states",
L. De Leo, C. Kollath, A. Georges, M. Ferrero, and O. Parcollet,
arXiv:0807.0790v2.

Recommended with a commentary by Thierry Giamarchi, University of Geneva

Understanding strong correlation effects remains one of the main challenges in condensed matter. In addition to the pure theoretical interest, incentives from experiments are many. Of of the most challenging one is of course the high temperature superconductors, which are suspected to be the results of strong correlation effects.

However, despite an arsenal of methods ranging from many-body theories of various sort to numerical approaches, we still lack a *good* method to tackle this problem. This is specially true in two dimensions, which is somewhat the worse situation. Indeed from the analytic front we lack the successful methods working well in one dimension, and the various mean field approaches, which are useful in high dimension deteriorate as the dimension gets smaller. On the numerical front, the sign problem which is deeply rooted in the fermionic nature of the system and the antisymetrization of the wavefunction, is for the moment a barrier that numerical methods cannot pass for low temperatures and/or large systems.

Even the simplest of the interacting models, the "Ising model" of strong correlations, the Hubbard model, where the electrons hop between the sites of a lattice with an amplitude t and interact only with an on-site interaction U cannot be solved. Or rather, it has been solved in so many ways, and with so many uncontrolled approximations or extrapolation of numerical calculations that it is difficult to know how much is due to the approximation itself and how much is real. Experiments in condensed matter are of little value to understand this point, since of course real life is much more complicated than the Hubbard model (long range interactions, more than one orbital per site, other degrees of freedom etc.). It is thus difficult to know whether the phenomenon that one struggles to find in the Hubbard model (e.g. d-wave superconductivity for repulsive interactions in two dimensions) is really there in such a model, or whether it is due to an extra ingredient.

In that respect cold atomic systems have provided an interesting alternative route to tackle this question. Indeed using light it is possible to realize optical lattices in which fermions (such as ^{40}K atoms) can be trapped. The fermions hop between the sites of this lattice, whose height (and thus hopping amplitude t) can be controlled at will. Atoms being neutral they interact only when they are on the same site of the lattice, making it a perfect realization of the Hubbard model (for a review on optical lattices see 1. above.). In addition to the control over the kinetic energy and hopping amplitude, the interaction U itself can be changed at will using a Feshbach resonance. Cold atomic fermions in optical lattices are thus a very promising tool to study the Hubbard model physics in a very controlled situation, and act as "quantum simulators" for this problem.

In the second and the third paper recommended above, an important step in this direction was reported. The teams in Zurich and in Mainz report to have observed the incompressible Mott insulating phase corresponding to large interactions and half filling for the Hubbard model on a three dimensional cubic lattice. Although in itself the half-filled case of the Hubbard model is not a very challenging issue, since we know for long that it should give an antiferromagnetic insulator, this is the first step in the direction outlined above. Two points are worth noting:

1. The system is in a confining trap, leading to a non uniform chemical potential $\sum_i \frac{1}{2}\omega_0 R_i^2 n_i$ where R_i is the distance to center of the trap. For studying the half filled incompressible insulator, this potential is a blessing. It allows to fill the lattice with one particle per site close to the center of the trap, regardless of the precise number of fermions that have been loaded in the trap. On the other hand it leads to non homogeneous states that are more difficult to understand and measure, and which are clearly a great complication in analyzing the physics of

these systems.

2. The temperature is still quite high for fermionic systems due to the difficulty in cooling fermions. It corresponds roughly to $T = T_F/6$ which by condensed matter standards would correspond to a very high temperature (of the order of $2000K$ or so !).

In the above systems, the Mott phase can be probed by various techniques such as shaking the lattice and measuring the double occupancy as a result of this shaking, or by varying the external trap, squeezing the system and measuring the radius to check for incompressibility. The complexity to determine if one has reached a Mott state is of course directly related to the non-homogeneity of the system. I refer the reader to the two papers [**?**, **?**] for more details on the measurements used. To analyze the results, given the high temperatures, and the control over the parameters it is possible to make very useful and *controlled* comparisons with numerical techniques, in the regime where these techniques are reliable. For example see the first and the fourth paper suggested above for a comparison with DMFT. This allows to analyze quantitatively the physics measured in these systems, in connection with Mott physics. Of course this will serve as a strong basis for when the system enters a regime of parameters (low temperatures and/or low dimensions) for which the numerical techniques become unreliable, but which is much more interesting in respect with the physics.

To go from the question of the Mott phase at half filling to whether the doped 2D Hubbbard model is superconducting or not there is a long way, with various important points:

- Changing the dimensionality is easy, since the tunnelling amplitudes can be controlled at will. This is already done "routinely".

- Cooling further is a major issue. This is mostly an experimental question, but of course it is worth thinking if new interesting proposals could help. On a more theoretical point of view, the question of thermometry is an important one, since one wants to have access to the temperature inside the fermionic system. What is the good quantity or procedure to measure this is an important and still largely open question.

- Measuring more complex orders such as antiferromagnetism or superconductivity is not easy. More generally the question of probing in cold atomic systems is an important and very challenging question both from the experimental and theoretical point of view. There are of course zillions of proposals on how to do this in theory but what will count is what is possible in practice. The inhomogeneity of the system (due to the trap) is not helping in that respect, so to access properties that are crucially dependent on the doping local probes will most likely prove an invaluable tool.

- If for the half filled case the trap is a blessing it is of course a curse when one comes to the question of doping the system (unless one can locally access the rings of "doped" material around the incompressible regions). Finding good methods to solve this question is of course a challenging issue

So cold atoms in optical lattices are moving along the yellow brick road, on their way to serve as an extremely useful tool for condensed matter systems and strong correlations. It is too early to know whether the wizard will be indeed at the end of the road, but the trip itself is challenging enough to be stimulating both for experimentalists and theorists.