Experimental Search for Non-Abelian Anyons

“Measurement of filling factor 5/2 quasiparticle interference with observation of charge $e/4$ and $e/2$ period oscillations”

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The most novel and striking feature of the theory of the fractional quantum Hall effect is its prediction of particles with exotic braiding statistics, or *anyons*. Ironically, this is the one key feature of the theory which has yet to be directly and unambiguously observed experimentally.

In three dimension, particles must be either bosons or fermions. However, in two dimensions, other possibilities exist since two counter-clockwise exchanges cannot be adiabatically deformed into zero exchanges. Consequently, the “statistical phase,” $e^{i\alpha}$, which the quantum-mechanical wavefunction acquires upon interchange is not required to be ± 1. Indeed, such particles are predicted to exist in the new states of matter which have been discovered in the quantum Hall regime, starting in the early 80’s and continuing to today.

Experiments which directly measure anyon statistics are not easy. Many effects cause a phase shift of the wave-function, so that isolating the statistical contribution (proportional to α) is challenging. To get a sense of the difficulty, consider a two point contact interferometer in a quantum Hall state supporting a quasiparticle with charge $e^\star = e/m$ and statistical angle $\alpha = \pi/m$. As the magnetic field B is varied, the current through the interferometer oscillates reflecting the relative phase shift of the portions of the wave-function scattered from the first and second point contact. There are two “geometric” contributions to this phase shift - an Aharonov-Bohm contribution, $\Delta\theta_{AB} = 2\pi AB/\Phi_0^\star$, with $\Phi_0^\star = hc/e^\star$, and a statistical angle, $\Delta\theta_{stat} = 2n\alpha$, where A and n are, respectively, the area and the number of localized quasiparticles enclosed by the quasiparticle trajectory. The problem is that changing B or varying A through variation of the voltage applied to a side gate can also lead to changes in n, so that it is difficult to differentiate the phase shifts arising from the fractional charge and the fractional statistics of the current-carrying quasiparticles. A successful experiment should, therefore, control separately the area of the interference loop and the number of quasiparticles contained in it as the magnetic field and sidegate voltage are varied. It has, thus, proven to be very difficult to directly observe the fractional statistics of quantum Hall quasiparticles. As a further indication of the difficulty involved, we note that a 2005 attempt by Camino *et al* [1] is still the subject of debate.

In the quantum theory of identical particles in two dimensions, a still more exotic possibility is that the particles have "non-Abelian" statistics. This arises when, even after the positions and local quantum numbers of a collection of particles have been specified, there is an N-fold degenerate set of states, separated from all other states by an energy gap ΔE , so long as the particles are kept further apart than a characteristic length scale ξ. Then, at temperatures much smaller than ΔE, taking one particle adiabatically around a loop encircling some of the others will transform an initial state ψ_a into a linear superposition of degenerate states, $U_{ab}\psi_b$, where U is an $N \times N$ unitary matrix which depends only on the braiding topology of the loop. Taking a particle (either the same one or a different one) around another trajectory will transform the state into $U'_{ab}\psi_b$. If the matrices U and U' do not commute, at least for some pairs of particle trajectories, then we say that the particles are non-Abelian anyons. Note that different degenerate states ψ_a and ψ_b cannot be distinguished by local quantum numbers. This also means that the degeneracy is stable against local perturbations – a fact which suggests possible applications to quantum computing.

There are compelling theoretical indications that the observed $\nu = 5/2$ fractional quantum Hall state supports quasiparticle excitations which have charge $e^\star = e/4$ and are non-Abelian anyons. Specifically, their statistics is related mathematically, through a formulation in terms of Majorana fermions, to the critical 2D Ising model and to chiral p-wave superconductors and are, therefore, often called Ising anyons. Previous experiments have produced suggestive evidence of charge $e/4$ quasiparticles. However, no experiment – until now – has directly probed the braiding statistics of the quasiparticles in the $\nu = 5/2$ state.

Ironically, non-Abelian statistics may be easier to observe than Abelian ones. This is nowhere more striking than in the case of Ising anyons. In their putative incarnation in the $\nu = 5/2$ state, particles whose charges are an even multiple of $e/4$ (i.e. carrying an integer number of flux quanta) are Abelian while those which are an odd multiple of $e/4$ (carrying a half-integer number of flux quanta) are non-Abelian. Consequently, when there is an even number of $e/4$ quasiparticles localized in the interferometry loop, current-carrying $e/4$ quasiparticles going around the interferometer will perform ordinary Abelian braiding and an interference pattern will be observed (as a function of magnetic field or area). However, if there is an odd number of localized $e/4$ quasiparticles in the interference loop, current-carrying $e/4$ quasiparticles going around the interferometer will perform non-Abelian braiding and the topological state of the quasiparticles will record which of the two routes the current-carrying $e/4$ quasiparticle takes. The special feature of Ising anyons is that the two topological states corresponding to the two possible routes are orthogonal. Thus, current-carrying $e/4$ quasiparticles taking different routes around the interferometer cannot interfere. Note, however, this does not mean that there will be no interference pattern whatsoever. Charge $e/2$ quasiparticles, which can be viewed as a pair of $e/4$ quasiparticles, are Abelian and will also give a (presumably subleading) contribution to the current around the interferometer. These will give rise to interference effects regardless of whether there are an even or an odd number of quasiparticles enclosed.

Willett *et al.* have constructed a two-point contact interferometer in a high-mobility GaAs device in which the $\nu = 5/2$ state is visible. By varying the voltage on a sidegate, they change the area of the region between two constrictions. As the sidegate voltage is varied, the current through the interferometer varies. The change in area is assumed to be proportional to the change in sidegate voltage, and the constant of proportionality is assumed to be independent of filling fraction. They further assume that the period at $\nu = 2$ corresponds to a change in flux of Φ_0. These assumptions lead them to conclude that the period at $\nu = 5/3$ and $7/3$ is $\approx 3\Phi_0$. If the number of quasiparticles in the interference loop doesn't change as the area is varied (at least for small variations), then this period implies that the current is carried by charge $e/3$ quasiparticles. The data show regions of regular oscillations separated by 'glitches' in which no clear periodicity is apparent. These may be the phase slips caused by the motion of quasiparticles in or out of the interference loop or, perhaps, regions in which the area does not change regularly with sidegate voltage. At $\nu = 5/2$, there are regions in which the period is $\approx 4\Phi_0$ and other regions in which it is $\approx 2\Phi_0$. 'Glitches' separate the two types of regions. Willett *et al.* interpret their two regions as corresponding to even and odd numbers of enclosed quasiparticles. When the quasiparticle number is even, oscillations with period $4\Phi_0$ are observed. When it is odd, these oscillations will not be observed, as described above. Thus, only period $2\Phi_0$ oscillations will be observed. The 'glitches' are those values of the sidegate voltage for which a quasiparticle is entering or exiting the interference loop. If correct, this would be striking confirmation that charge $e/4$ quasiparticles are non-Abelian in the $\nu = 5/2$ state.

When there are only a few oscillations in a row, determining the period of the oscillations is not entirely trivial. However, Willett et al. make a strong case using Fourier transforms of runs of the data containing both $e/4$ and $e/2$ regions and also Fourier transforms of the $e/4$ and $e/2$ regions separately. Similar Fourier transforms at $\nu = 2$, $\nu = 5/3$, and $\nu = 7/3$ indicate that the constant of proportionality between the sidegate voltage and the area is, in fact, a constant. However, there are still reasons why one might be skeptical. If the $e/4$ and $e/2$ really correspond to even and odd quasiparticle numbers, then half the regions should be $e/4$ and half $e/2$. Furthermore, the amplitude of $e/2$ oscillations should be the same in both $e/4$ and $e/2$ regions. It is not clear that either is the case. Nevertheless, these experiments mark a quantum leap towards directly observing non-Abelian anyons – a new type of particle never before seen.

[1] F. E. Camino, W. Zhou, and V. J. Goldman, Phys. Rev. Lett. **72**, 075342 (2005).