Superconductivity in the repulsive Hubbard model: an asymptotically exact weak-coupling solution

Authors: S. Raghu. S. A. Kivelson and D. J. Scalapino

arXiv:1002591v1 [cond-mat.supr-con]

Recommended and Commentary by Peter Wölfle, Karlsruhe Institute of Technology, Germany

The existence and nature of superconductivity (or superfluidity) of Fermi systems interacting (primarily) via repulsive interactions has been a long-standing subject, ever since BCS-theory was proposed. The pioneering works of Kohn and Luttinger (1965), Berk and Schrieffer (1966), Anderson and Brinkman(1975) explained correctly how an attractive effective interaction may appear out of a primary repulsive interaction. Qualitatively, the repulsive interaction generates fluctuations in the system, which may mediate an attractive interaction just like phonons do in the standard BCS model. The most widely discussed fluctuations in this context are spin fluctuations of either ferromagnetic or antiferromagnetic character (K. Miyake, S. Schmitt-Rink and C. M. Varma, Phys. Rev. B 34, 6554 (1986)). Depending on the system, nonmagnetic fluctuations may be of importance, too. For example, in liquid He3, an almost localized system (which may be thought to be close to a Mott transition), one finds strong transverse current fluctuations in addition to ferromagnetic spin fluctuations. An analysis of the effective interaction in He3 in the framework of Fermi liquid theory, making use of all available experimental information (thermodynamic, transport, relaxation properties) allows to identify p-wave pairing of reasonable strength as the most stable channel (M. Pfitzner and P. Wölfle, J. Low Temp. Phys. 51, 535 (1983)). However, the range of the interaction in energy and therefore the transition temperature, can only be estimated. A comprehensive review of mechanisms of superconductivity has been recently given by Varma (C. M. Varma, arXiv:1001.3618 [cond-mat.supr-con]).

The situation is even more complex in the case of high temperature cuprate superconductors, since the pair condensation there happens out of a "strange metal" or pseudogap phase. Attempts to derive superconductivity on the basis of Hubbard-type models are con-

troversial. It is therefore legitimate to ask the question: What do we know for sure about superconductivity in repulsive lattice models? This question is answered in the limit of weak coupling in the above paper by Raghu, Kivelson and Scalapino . They employ a two-step renormalization group method to identify the channel showing strongest attraction and to calculate the characteristic energy, which is essentially the transition temperature. This is done in an expansion in the small Hubbard U, to second lowest order. Results are presented for a variety of band structures ($t - t'$-hopping) and lattices in two and three dimensions. For example, on the two-dimensional square lattice for densities $n = 0.6$ to 1.4 , and in the range of next nearest neighbor hopping amplitudes $t'/t = 0$ to -0.3 the $d_{x^2-y^2}$ pairing is by far strongest. On a 3d bcc-lattice it is found that at low density the three dimensional p-wave T_{1u} is dominant, while at higher densities the d-wave E_g and near to $n = 1$ the T_{2g} representation wins. In the cases considered a large variety of behavior is found. The highest pairing strength (about a factor 2 higher than for the 2d square lattice Hubbard model) is obtained in the case of spin triplet pairing of B_{1u} symmetry on the 2d honeycomb lattice near $n = 1$.

Estimates of the critical temperature on the basis of results presented in the paper yield values of the dominant coefficient α_2 in the T_c-formula $T_c = W \exp[-\alpha_2(t/U)^2 - \alpha_1(t/U) - \alpha_0]$ of $\alpha_2 \approx 30$ for d-wave pairing on the square lattice with $t'/t = -0.3$ and near filling $n = 1$ and $\alpha_2 \approx 15$ for B_{1u}-triplet pairing on the honeycomb lattice near $n = 1$. The values of T_c/W at $U/t = 1$ are correspondingly 10^{-6} and 10^{-3} in the two cases, respectively. Since these are among the most favorable cases reported, it is seen that the T_c values obtained are generally rather low.

The two-step RG procedure used introduces a fictitious energy scale Ω_0 separating the two steps. It is shown how this scale drops out of the final result, once certain second order correction terms in the effective action are kept. A full functional renormalization group treatment of the problem (along the lines of their ref.[28]) avoids the artificial separation of scales and provides in principle a quantitative determination of T_c . However, it would be by far more costly in numerical effort.

The value of the analysis presented by Raghu, Kivelson and Scalapino lies in its being (1) rather generally applicable (extensions to finite range interactions, more complex single bandstructures, multiband systems are possible) , (2) unbiased and complete (3) quantitative as far as the exponent in the T_c-expression is concerned.