“Maximum and Minimum Stable Random Packings of Platonic Solids”

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Packing problems are extremely easy to pose but notoriously difficult to solve rigorously. For example, how densely can nonoverlapping particles fill d-dimensional Euclidean space and what is the corresponding arrangement [1]? Although Kepler conjectured the densest packing of identical three-dimensional spheres almost over four centuries ago, it took nearly that amount of time to prove that the face-centered-cubic lattice (the way your green grocer stacks oranges) is indeed the densest arrangement [2]. Packing problems are ancient but they continue to provide fascinating challenges for scientists and mathematicians [1,2,3,4].

Dense packings of hard particles have served as useful models to understand the structure of low-temperature phases of matter, granular media, heterogeneous materials, and biological media [3]. Disordered jammed sphere packings have been employed to understand glassy states of matter [3,4]. There has been a resurgence of interest in maximally dense sphere packings in high-dimensional Euclidean spaces [1,3,5]. Interestingly, the optimal ways of sending digital signals over noisy channels correspond to the densest sphere packings in high-dimensional spaces [1,3].

It is only very recently that attention has turned to understanding dense packings of congruent nonspherical particles in three-dimensional Euclidean space, including ellipsoids [6], superballs [7], tetrahedra [8-12], and all of the Platonic and Archimedean polyhedron solids [9]. It was conjectured that the densest packings of the centrally symmetric Platonic and Archimedean solids are given by their corresponding optimal lattice packings [9], which is the analog of Kepler's sphere conjecture for these polyhedra.

An interesting fundamental and practical question is how polyhedron solids randomly pack? Baker and Kudrolli have carried out an interesting series of experiments to understand random packings of polyhedral particles. Specifically, they experimentally investigated finite but dense random jammed packings of Platonic-solid-like plastic dice and tetrahedron-like ceramic particles. These packings were produced either by fluidization or vibration of particle containers in order to achieve mechanically stable states under gravity. In particular, dense random jammed states were obtained through vibrations of containers of particles and so-called “random loose” packings were produced by randomly and sequentially adding particles in a container. The authors measured the densities of the packings of the Platonic-like solids and found that the density peaks for the cube-like shape, and monotonically decreases as the number of faces of the particle increases. A similar trend was also observed in maximally dense packings of frictionless ideal Platonic solids [9].

A notable difference was observed in the measured packing fraction for random jammed packings of the tetrahedron-like dice in the current study (0.64) compared to that carried out by Jaoshvili *et al.* (0.76) [13]. To explain this large discrepancy, the authors systematically studied the effects of friction on the packing density using tetrahedron-like particles made of different materials. The authors claimed that their tetrahedron-like dice contained a higher degree of friction compared to that in the tetrahedron-like dice packings investigated by Jaoshvili *et al.* [13], and thus their packings should have a much lower density. It would also be interesting to see to what extent the particular packing protocols and density-measuring methods used by the two groups contributed to the density discrepancy. The present authors also made an attempt to characterize the degree of disorder of the packings by measuring the projected areas of the largest visible particle face in the top layer of the packing. It would have been interesting if the authors could have provided additional packing characteristics, such as the mean contact number, pair correlation functions, and orientational correlation functions.

This work raises many interesting issues and questions for theorists. We are very far from a complete theory of random packings of spheres, let alone for polyhedral particles. An obvious question is how to generalize the well-studied methods for quantifying order/disorder in sphere packings to disordered polyhedron packings. What are the configurations corresponding to the maximally random jammed states and lowest density jammed states [3] of polyhedron packings? Jaoshvili *et al.* [13] showed that their random packings of tetrahedron-like plastic dice are essentially frictionless and *isostatic* (minimal number of interparticle contacts) in a *generalized* sense. However, it is not clear whether packings of other polyhedron particles (using the protocol of Ref. [13]) are also isostatic in this generalized sense. Can one define mathematically precise jamming categories for packings of polyhedra and other nonspherical particles? Another important issue is understanding the effect of friction in polyhedron packings, which apparently is more significant than that found in sphere packings (for the same particle material) due to the larger contact regions between the particles. Again, there are many challenges that lay before us that must be overcome to make progress on a comprehensive theory of random packings of nonspherical particles.

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