Commentary on "Spin-Orbit coupled Bose-Einstein condensates", by Y.-J. Lin, K. Jiménez-García; & I. B. Spielman. Nature 471, 83-86 (2011).

by Tin-Lun Ho

Slightly over a year ago, Ian Spielman's group at NIST succeeded in producing an artificial electron-magnetic field for a Bose-Einstein condensate (BEC) through a simple experimental setup, (Y.-J. Lin, R. L. Compton, K. Jiménez-Garcia, J. V. Porto, I. B. Spielman, Nature 462, 628 (2009). The "magnetic field" produced was strong enough to generate a vortex array. None of the delicate adjustments crucial for the usual rotating gas experiments are needed. This work has generated great excitement for it shows a new way to reach high angular momentum regime. It has provided new impetus to the effort to realize quantum Hall states in cold atoms.

Very recently, Spielman's group has reported the creation of spin-orbit interactions in a Bose-Einstein condensate, using the *same* setup as their work in 2009, but in a different parameter regime. This is a significant development, for it immediately opens many new doors for quantum simulation with cold atoms. At the same time, it provides opportunities to study spin-orbit phenomena that are unique to cold atoms because of their rich internal degrees of freedom. The spin-orbit effects reported in the paper above are such examples.

The fact that "electromagnetic" field (which is abelian) and spin-orbit interaction (which can be made either abelian or non-abelian) emerge in different parameter regimes of the same experimental set up means they are simply members of a much larger family of gauge fields. This is how it works. Consider a Hamiltonian $\hat{H} = \vec{p}^2/2M + \hat{W}$, where $\hat{W}$ is a matrix in spin-space and *is spatially varying*. If $\hat{W}(\vec{r})$ has a *non-degenerate* ground state $|g\rangle_{\vec{r}}$ sufficiently *isolated* from all higher states, then the effective Hamiltonian restricted to the subspace of $\{|g\rangle_{\vec{r}}\}$ will acquire an abelian gauge field $\vec{A}(\vec{r}) = -i\hbar_{\vec{r}}\langle g|\nabla|g\rangle_{\vec{r}}$ in the kinetic energy, which assumes the form $(\vec{p}+\vec{A})^2/2M$. On the other hand, if the ground state of $\hat{W}$ is doubly degenerate (say, $|0\rangle_{\vec{r}}$ and $|1\rangle_{\vec{r}}$) and are isolated from all other higher states, then the gauge field in effective Hamiltonian is a matrix in spin space $\{|\alpha\rangle_{\vec{r}},\ \alpha = 0,1\}$, $\vec{A}_{\alpha\beta}(\vec{r}) = -i\hbar_{\vec{r}}\langle\alpha|\nabla|\beta\rangle_{\vec{r}}$; and the term $\vec{p}\bullet\vec{A}$ in the Hamiltonian immediately implies spin-orbit interaction.

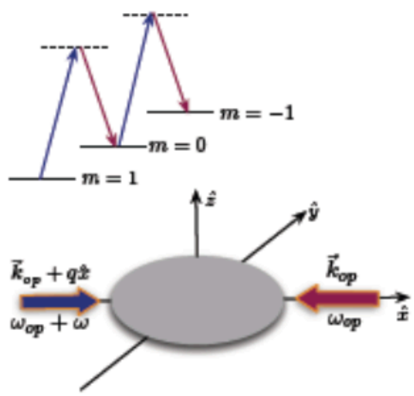
This is what happened in the NIST experiments, where two lasers with different frequencies and polarizations are directed toward a BEC along $\hat{x}$ and $-\hat{x}$ in the presence of a non-uniform magnetic field $\vec{B}$ along y. (See Figure 1) The effect of this arrangement is to impart a net momentum $k_L\hat{x}$ to an atom while flipping its spin along $\hat{y}$. Rather than going into the details of the effective Hamiltonian, we simply describe its energy spectrum and wavefunction, which is all we need for our discussions. The eigenstates of this system are two component spinors, as the lasers operate on two special hyperfine states. They are labeled by a momentum q and of the form $e^{iqx}\zeta_\mu(q)$. An example of the single particle spectrum is shown in the red curve in Figure 1. It consists of two branches separated by a gap, and that the lower branch has two degenerate minima, say, at $k_1\hat{x}$ and $k_2\hat{x}$. This is the spin-orbit coupled case we discussed before. Due to the degeneracy of these two states, the condensate wavefunction is a linear combination of them, $\Psi_{SO}(x) \sim \sqrt{N/2}[e^{ik_1x}\zeta_\mu(k_1) + e^{ik_2x}\zeta_\mu(k_2)]$. That the BEC indeed contains these momentum states has been revealed in the time of flight experiments, which is a clear demonstration of the spin-orbit effect in the system.

As the parameters of the system are changed, the lower branch in Figure 1 can evolve from a double minima to a single minimum, (which are shown by the sequence of lower curves in Figure 1), thereby reducing to the abelian case previously studied, and the vortex array is recovered.

Without questions, there will be a lot more to come in the study of spin-orbit effects in cold atoms. The obvious direction is to study fermions, and the exploration of strong correlations effects. Yet even in the case of bosons, spin-orbit interaction has brought forth some great possibilities. The fact that $\Psi_{SO}(x)$ is a linear combination of momentum states means it is a density wave. The BEC therefore acts like a periodic potential for other types of atoms. However, unlike optical lattices, which have no dynamics of their own, these matter lattices have phonons. They therefore allow exploration of phonon related problems in solids using cold atoms.

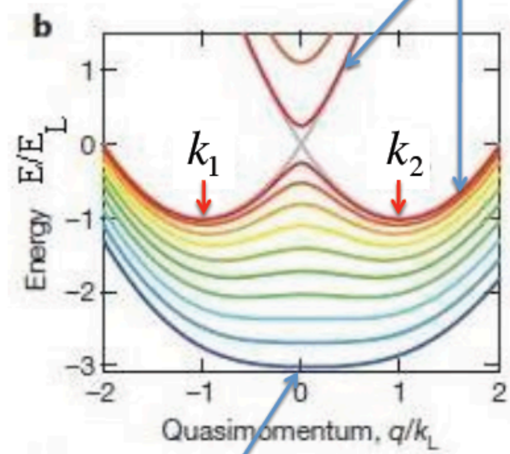
The new directions are numerous indeed.

Figure 1



Experimental set up of Y.J. Lin et.al.

Energy spectrum in spin-orbit regime.



Energy spectrum for the abelian case