Discretizing wrinkling

Elastic Building Blocks for Confined Sheets, R. D. Schroll, E. Katifori, and B. Davidovitch, Phys. Rev. Lett. **106** (2011) 074301.
and
Wrinkling hierarchy in constrained thin sheets from suspended graphene to curtains, M. P. H. Vandeparre, F. Brau, B. Roman, J. Bico, C. Gay, W. Bao, C. Ning Lau, P. M. Reis, P. Damman, arXiv:1012.4325v1 (2010).

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Thin elastic sheets are floppy. Therefore they can be easily deformed into 3D configurations. In some cases the bending deformation is "trivial" (think of a buckled ruler). In other cases, such as in wrinkling of skin, the configurations are "interesting", containing some typical length scale, much smaller than the system's size. Finally, in some cases, such as in crumpling, or wrinkling-cascades, the configurations are "surprising", consisting of multi-scales and often focussed stresses.

Beyond the direct context of elastic sheets, these *types* of solutions can be vied as representatives of general ways of organizing matter and energy. The buckled ruler is a "laminar" solution – it does not contain scales much different from the system's size with energy distributed over large length scales, the wrinkling is a "wavy" solution, with a single length scale for both configuration and energy distribution, the wrinkling cascade is a hierarchical type of organization, switching between length scales, and, finally, crumpling is a "defect dominated"/ or "turbulent" organization, consisting of a continuum of scales with strong focusing of energy. All this variety of behaviors stems from the known, simple elastic energy functional of thin sheets, with only geometry and boundary-conditions selecting between them. It is therefore tempting to use thin elastic sheets as a model system from which we learn how geometry leads to different *types* of solutions in more complex fields. For example, the conditions for having or not having a crumpling solution are directly linked to the geometrical confinement problem [1], while the type of configurations of non-uniformly growing sheets are governed by an underlying embedding problem [2].

The apparent generality of thin sheets instabilities, mentioned above, led physicists to ask questions about this very "engineering system": Is it possible to identify more general principles that govern shape selection "strategies" in thin sheets? Is classification of types of patterns useful and meaningful?

The study of crumpling represents a relatively successful such a research, showing the existence of "building blocks" of the global configuration. These are "d-cones" and "ridges". The energy within such isolated "objects" was studied and properties of global crumpled state were predicted as ensembles of such elementary objects [3]. Such progress was absolutely impossible using only the basic elasticity field equations.

Two recent works have taken the first steps performing a similar treatment for multi-scale, or hierarchical wrinkling patterns. Such patterns are general, appearing along the edge of suspended films, curtains, blistering of delaminated films, and growing sheets. They consist of repeating "motifs" in different scales, suggesting that, similarly to crumpling, it could be worthwhile describing the global patterns as collections of elementary units. The basic motif is the splitting of a single out of plane buckled node of width/wavelength W into two nodes, each of width W/2. The work by Schroll, Katifori and Davidovitch [4] is a theoretical and numerical study of such a single transition. The authors generate such an object (using specific boundary conditions) and study its energy distribution, shape and dominant length scales. They find two distinct domains: A stress-focusing, crumpling-like “core”, connected to a smooth “diffusive” zone in which energy is spread over a large area. The diffusive zone appears when the in-plane stress is dominated by a single tensile component.

The authors find the typical scale over which the energy is spread (and the transition takes place) and show that such an “object” is more energetic than the stress-focusing structures that dominate crumpling. They suggest that it is likely that the new object is the “second best” option, appearing when boundary conditions exclude the existence of the low energy crumpling-like structures.

A similar system was studied jointly by several European and US groups [5]This study was both experimental and theoretical (scaling arguments). The main emphasis in the work was the appearance of the same type of pattern in many systems, over an amazingly large range of scales, starting with confined Graphene sheets, up to meter-scale curtains. As in the work by Schroll, *et al.*, they have defined the elementary structure naming it a “wrinklon”, obtained an expression (identical to that calculated by Schroll) for its energy content and went further, calculating properties of a *global* hierarchical wrinkling pattern by a “superposition” of wrinklons. The work shows an impressive agreement between experimental data and predictions. It also demonstrates that the same treatment is applicable for sheets under a different stress state. This is shown for “heavy sheets” in which gravitation leads to different scaling then in the “light sheets”.

The works reviewed here represent a new trend in soft matter physics, which focuses on studying new general “surprising” cases of elastic instabilities. There is a truthful attempt not to stay in the demonstrative level, but to link the results to “real systems”. The scale-free nature of the elastic field allows mapping results between systems of different scales. Therefore, results obtained from experiments on macroscopic scale are shown to be relevant to instabilities of Graphene sheets, or useful in constructing materials with unique mechanical properties (some nice examples can be found in Soft Matter 6, 2010). Similarly, the role of such instabilities in the evolution of biological systems is studied extensively. The results reported in the reviewed papers are, therefore, likely to be relevant to solid state physics, material science and biology.

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[3] T. A. Witten, Rev. Modern Physics **79**, 634 (2007).

[4] R. D. Schroll, E. Katifori, and B. Davidovitch, Phys. Rev. Lett. **106**.

[5] M. P. H. Vandeparre, F. Brau, B. Roman, J. Bico, C. Gay, W. Bao, C. Ning Lau, P. M. Reis, P. Damman, arXiv:1012.4325v1 [cond-mat.mtrl-sci] (2010).