Is glassy physics relevant to superconductor-insulator transition?

Disorder-driven quantum phase transition in superconductors and magnets, L.B. Ioffe and M.Mézard, Phys.Rev.Lett. 105, 037001 (2010)
Superconductor-insulator transition and energy localization, M.V. Feigel'man, L.B. Ioffe and M. Mézard, Phys. Rev.B 82, 184534 (2010)

Recommended with a Commentary by Claudio Castellani, Universita' di Roma "La Sapienza"

The two papers by Ioffe and Mézard and by Feigel'man, Ioffe and Mézard raise the intriguing question whether the glassy physics can be of relevance for the superconductor-insulator transition (SIT) in two-dimensional disordered films. In the last years there has been a great advance in our knowledge of strongly disordered superconductors from the experimental point of view. Experiments of scanning tunneling microscopy (STM) on various materials like films of InO, TiN, and NbN [1–5] have definitively established that in these systems superconductivity disappears at strong disorder without closing the density-of-states (DOS) gap. There are Cooper pairs above T_c and superconductivity is destroyed by loss of coherence: the superconducting order parameter disappears but not the pairing gap. Theoretically this is an old suggestion (see for instance Ghosal, Randeria and Trivedi [6], but also the seminal paper by Ma and Lee [7]). However to see a STM map of the local gap (where the gap is changing from site to site, but not very much) provides a different feeling of confidence. In this kind of systems the zero-temperature quantum SIT can be tuned by applying a magnetic field which produces a characteristic giant positive magneto-resistance which then turns down at higher fields. In the insulator the resistance is activated with a typical energy T_0 which goes to zero at the transition. It is natural to think of this behavior as that of localized bosons with a finite mobility edge vanishing at the transition. (The list of relevant experimental evidences is really huge and one should also include the study of the SIT on artificial honeycomb substrates [8,9]).

What do the papers I'm recommending add to this picture of the SIT as a transition from coherent (superfluid) bosons to incoherent (localized) bosons? They add the claim that this happens in a glassy phase (glassy in the sense of the spin glass community, with replica symmetry breaking). They start to model the SIT with a XY model in a random Z-field (i.e. a tight-binding hard core boson system with on site disorder). Indeed they study the Ising version of the model, i.e. the Ising model in a random transverse field.
Even though the U(1) symmetry is lost, I think the main mechanism of competition between localization and coherent long range order is still present. They analyze the model on a Cayley tree by a cavity method, leading to (quite simple, mean-field-like) recursive equations. The surprise is that at low temperature the system looses self-averaging properties and the susceptibility for establishing coherence is dominated by few paths along the Cayley tree. In technical terms there is a one-step replica symmetry breaking and at low temperature superconductivity takes place in this phase. In less technical terms, self-generated and sample-dependent inhomogeneity takes places on a mesoscopic scale much larger than disorder modulation or the standard superconducting coherence length.

They solve the recursive equations by using a solution of Derrida and Spohn [10] for the Directed Polymer (DP) with quenched disorder. In its low temperature pinned phase (replica symmetry broken phase) the Directed Polymer with quenched disorder is a random walk which does not visit all possible sites because of local disorder. There are a few preferred paths, which are the counterpart of the mesososcopic inhomogeneity of Ioffe-Mézard picture of disordered superconductors. One of the consequences of the Ioffe-Mézard picture (or, one can say, of the mapping to the DP) is that the local superconducting order parameter (OP) has an anomalous distribution with huge tails. Indeed the recent STM experiment [3] shows that this is the case, by reasonably taking the height of the coherent peaks of the local DOS as a measure of local OP.

Applying the same recursive equations method to the dynamics in the insulating phase they claim that there is a finite mobility edge for the bosonic excitations which increases by moving inside the insulator and is vanishes at the transition. Personally I have doubts about this result, which is obtained at half-filling (half boson per site on average), while I expect that to be true away from the half-filling condition.

The mapping to Directed Polymer seems to be a quite powerful and intriguing tool. A main issue is that the Directed Polymer on a Cayley tree (the case studied by our authors, both in statics and dynamics) behaves differently (quantitatively and, under various aspects, also qualitatively) from the DP in two dimensions. So there is a lot to do in statics and in dynamics, specifically in two dimensions (indeed we are working on that in Rome).
A possible view of their results in two-dimensional systems is that superconductivity is maintained by sample-depending filamentary structures and the rest is proximity effect. This would have still unexplored consequences on many aspects, from vortex motion and critical currents to dissipation (indeed, specifically in the insulating phase), quantum critical behavior and so on. However, it could also happen that the filamentary structure will be washed out by the Goldstone mode related to the U(1) symmetry. As a theorist really I would like that the main claim of Ioffe and Mézard to survive and be supported by future experimental verifications.

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