Reynolds Pressure and Relaxation in a Sheared Granular System

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Recommendation and commentary by Bulbul Chakraborty

Solids are defined by their ability to resist shear. In the classical picture, solids respond elastically to small shear stresses but undergo plastic failure when the shear stress exceeds an yield stress. Soft solids such as foams, emulsions and colloids have intriguing rheological properties characterized by slow, glassy relaxations that are attributed to structural disorder and the presence of metastable states. These materials flow under imposed stress. Much recent work has focused on understanding the shear-induced flow in such materials through the frameworks of soft-glassy-rheology and shear-transformation-zones.

A very different scenario exists in athermal systems such as granular materials or concentrated, non-Brownian suspensions. It has been known for a long time that shear can lead to dramatically increased viscosities and jamming in such systems. Recent work by the groups of Daniel Bonn in Amsterdam, and Heinrich Jaeger in Chicago, have highlighted the role of granular physics in the dramatic rise of viscosities in athermal suspensions. An explicit realization of shear-jamming has recently been observed in a two-dimensional system of frictional particles [Refs. 1 & 2 of the paper]. The emergence of rigidity in these systems was shown to be related to the percolation of force chains. The nature of these shear-jammed states and their response to shear is, however, still pretty much unknown, and is the focus of the recent Physical Review Letter by Ren, Dijksman and Behringer.

Using an ingenious experimental set up, the authors create states that have uniform shear, thus avoiding complications that arise from the phenomenon of shear localization that occurs commonly in frictional materials. These states offer a unique insight into the nature of solids that are created by shear stress. The results should have broad-ranging implications since such shear-jammed states are expected to occur naturally in many granular systems and in dense suspensions that exhibit discontinuous shear thickening. The study of shear-jammed states also opens up a window into dilatancy (volume expansion under shearing), a well-documented but ill-understood phenomenon.

A remarkable finding of the experiments on shear-jammed states described in this paper is

the relationship between pressure (P) and shear strain (γ): $P = R\gamma^2$. The authors refer to this shear-induced pressure as the Reynolds pressure, drawing a connection to dilatancy. Since the experiments are carried out at fixed density, the sheared granular packing cannot dilate, and instead, shearing leads to increased pressure. This connection is not new and has been made in the context of discontinuous shear thickening, whose origin has also been traced to confinement frustrating the tendency to dilate. What is new about the current results is a quantitative measurement of the Reynolds pressure and characterization of the Reynolds coefficient R as a function of the packing fraction. The authors find that R diverges as the packing fraction ϕ approaches a critical packing faction ϕ_c. Numerically, the authors find that $\phi_c \approx \phi_J$, where ϕ_J is the packing fraction above which jammed states can exist at zero shear. This packing fraction is, therefore, reminiscent of Point J in the jamming phase diagram proposed by Liu and Nagel, which for frictional grains is now well established to be a critical point. For shear-jammed states of frictional grains, ϕ_J does seem to play a special role. The experiments, in fact, also identify a dynamical signature of this special point.

In cyclic shear experiments, the authors find that whatever strain cycle they impose, there is a relaxation to a limit cycle behavior in which the pressure follows the $P = R\gamma^2$ behavior, about some mean strain that is zero only for symmetric cycles. The coefficient R has the same form as described above. The natural "time scale" for relaxation, as measured by the number of cycles, n_0, has a form that is intriguingly similar to the Vogel-Fulcher form of relaxation times in supercooled liquids approaching the glass transition. I take the liberty of rewriting Eq. 3 from the paper to explicitly demonstrate this parallel:

$$n_0 \propto \exp\left(\frac{A}{(\phi_c - \phi)^{|\alpha|}}\right),$$

where $|\alpha| \approx 3.3$. The relaxation time, therefore diverges as Vogel-Fulcher at the critical packing fraction, $\phi_c \approx \phi_J$. The appearance of ϕ_J as a critical point in a "static" and a dynamic property of shear-jammed states is intriguing, to say the least. The experimental results strongly suggest the existence of some type of singularity at ϕ_J. As seen from Ref. 2 of the paper, there is a line in the stress-packing fraction plane that mark the onset of shear-jammed states. This line ends at ϕ_J. We, however, do not know if the line is a line of first-order transitions or a line of critical points. The end point of either would have special properties.

The asymmetric shear cycles begin with a pressure asymmetry, ΔP, between the beginning and the end of the cycle. This asymmetry, which depends on the packing fraction and the strain history,

relaxes to zero with the characteristic time scale , n_0, with a remarkable scaling form. As Fig. 4 of the paper shows, ΔP for different packing fractions and strain histories can be collapsed on to a universal scaling form, $\Delta P \propto -\beta \log(\frac{n}{n_0})$, where β is *only* a function of the strain amplitude. As the authors point out, this form suggests an activated process with β acting as an effective temperature and with ΔP, the barrier. The authors also point out that a pressure barrier and a stress-induced temperature are features that have been discussed in the context of the stress ensemble. The latter is a statistical ensemble approach to static and slowly driven granular materials. The experiments offer a unique opportunity and a challenge to these ensemble-based frameworks to explain the intriguing observations on shear-jammed states.

The carefully analyzed, and ingeniously executed experiments of Ren et al give us, theorists, a lot of food for thought and a hope for constructing a predictive theoretical framework. To the broader community, the experiments offer a detailed picture of the macroscopic properties of solids that are created by stress, an ubiquitous phenomenon in athermal systems ranging from grains to concentrated colloids to flocculated nanotube suspensions. While the thermal framework of matter has been explored for centuries, stress-induced solidification is in its infancy. Studies of this alternative mode of solidification, probing the nature of athermal condensed phases of matter and transitions between them, will deepen our understanding of condensed phases of matter in general.