Deep Understanding Achieved on the 3d Ising Model

Solving the 3d Ising Model with the Conformal Bootstrap II. c-Minimization and Precise Critical Exponents

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Recommended with a commentary by Leo P. Kadanoff , The University of Chicago and The Perimeter Institute

Over the years, the Ising model has yielded many important insights into statistical mechanics, specifically into phase transitions, the relation between liquid and gas, the behavior of alloys, and especially into the nature of critical points. This paper describes the critical behavior of the Ising model at three dimensions and thereabouts.

It is based upon techniques mostly developed by particle theorists: crossing symmetry, unitarity, conformal symmetry [1], as well as techniques common to field theory and condensed matter physics including operator algebra [2, 3], scaling, and universality. The result is a calculation, apparently of of unparalleled accuracy, of critical indices and the coefficients that describe the operator algebra. For example, the scaling length index ν is here calculated to have the value 0.62999(5), which is compared with the high temperature series [4] value 0.63012(16), and the experimental value [5] of 0.629+/-0.003.

Numerical accuracy is not the main virtue of this paper. This accuracy is the proof of the pudding. The nourishment, however, is that the numerical results are obtained from deep understanding of the structure of the Ising problem. Further the paper is so well-written that one can see and understand what is being done, even enabling one to see the parts that are most innovative.

Crossing symmetry and unitarity are used to calculate consistency conditions for two different expansions of the correlation function formed from four spins at criticality. The coefficients in the expansions are obtained in this and previous work using conformal symmetry and operator algebras. The consistency conditions combine with unitarity to produce a set of conditions in the form of inequalities. These inequalities are then combined with a condition that the three-dimensional Ising system occurs at the minimum conformal charge to then give a unique determination of the numerical and structural properties of this critical point.

The computation of the effect of many inequalities is achieved by developing a innovative application of linear programming techniques.

Important ideas about the structure of critical behavior are developed and illustrated by this paper. As in two dimensions [6], the three-dimensional critical behavior is part of a line of critical points parametrized by the conformal charge. The charge describes the distortion of space produced by the operators describing the critical point. As in two dimensions, the Ising charge is extremal because the Ising point is a place in which the line of critical points suddenly changes structure. One of the fluctuating operators, coupled to the critical point at charges greater than the Ising charge, becomes decoupled for lesser charges.

The success of this calculation suggests that similar calculations might give insight into the structure of other critical points and conformal theories.

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