Flow of transport coefficients in a film that shows the quantum anomalous Hall effect.

"Trajectory of Anomalous Hall Effect toward the Quantized State in a Ferromagnetic Topological Insulator", arXiv: 1406.7450

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Recommended with a Commentary by Bertrand I. Halperin, Harvard University

The observation of the quantum anomalous Hall effect in a ferromagnetic topological insulator, first reported by experimenters in Beijing last year[1], has attracted considerable attention, including a Journal Club Commentary by Leonid Glazman in July 2013. The new e-print by Checkelsky *et al*. reports experiments by a Japanese group, which further explore the properties of this fascinating system. In addition to confirming general results obtained by the Beijing group, the new work has mapped out the behavior of the longitudinal and Hall conductivities, as a function of temperature and gate voltage, and has found agreement with a picture of a renormalization group flow previously postulated for crossover behavior between quantized Hall states in the familiar integer quantum Hall effect. (See figure, below). This observation is quite interesting, as it is not clear, a priori, why transport properties should be described by such a renormalization group flow in the experimental regime.

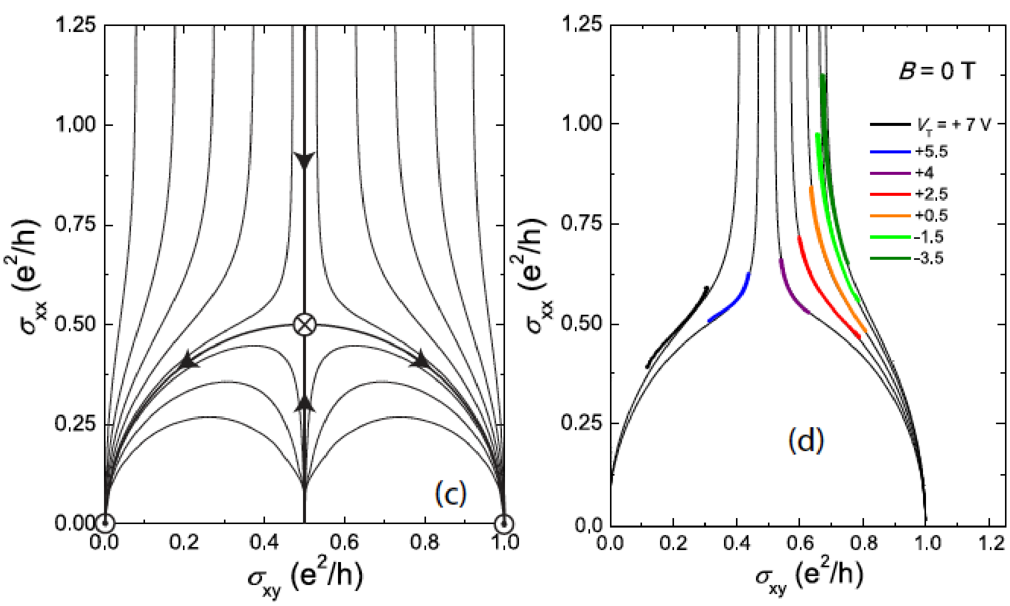

Figure Caption.

Right Panel. Colored curves show data for the Hall conductivity and diagonal conductivity, over a range of temperatures, for seven values of the top gate voltage V_T , for a sample with the maximum remnant magnetization, measured in zero magnetic field. Thin black curves are selected RG trajectories from the theory of B. F. Dolan, Ref. [2].

Left Panel. Fuller illustration of RG trajectories from Dolan's theory.

The conventional *anomalous* Hall effect, which has been known since the 19^{th} century, occurs in ferromagnetic materials, and is characterized by a non-zero Hall conductance in zero applied magnetic field. The observed Hall conductance is controlled by the remnant magnetization; it is a consequence of spin-orbit coupling, and it is not quantized. By contrast, the *quantized* Hall effects, observed over the last thirty years in non-magnetic two-dimensional semiconductor structures, arise from the orbital effects of an applied magnetic field. Now, the *quantum anomalous* Hall (QAH) effect has been observed, in effectively two-dimensional samples of a ferromagnetically doped topological insulator [$Cr_x (Bi_{1-y} Sb_y)_{2-x} Te_3$] with remnant magnetization, in zero applied field. Like the ordinary quantized Hall effect it leads to a Hall conductivity quantized in multiples of e^2/h, and, at least in principle, a longitudinal conductance that should vanish in the limit of zero temperature. Spin-orbit coupling is an essential ingredient for the QAH effect.

A useful property of two-dimensional systems is that their electron density may be varied by means of voltage applied to an external gate. If one varies the carrier density in a conventional quantized Hall sample over a sufficient range, at fixed magnetic field, one may be able to observe a transition between one quantized Hall state and another. (Transitions can also be induced by varying the magnetic field at fixed carrier density.) In very clean samples, at low temperatures, one may encounter a series of intermediate fractional quantized Hall states, or states with charge-density waves or other broken symmetry, between integer quantized Hall states. In less clean samples, one can observe direct transitions between integer states. As a special case, one might observe, in the limit T=0, a sharp transition between a quantized Hall state with Hall conductivity e^2/h, and an insulating state where the Hall and longitudinal conductivities are both zero.

It was proposed, in the 1980s, that the transition between integer quantized Hall states, at non-zero temperature, can be described by renormalization group (RG) equations involving the longitudinal and Hall conductivities, σ_{xx} and σ_{xy} , which we express here in units of e^2/h. Length scales in a macroscopic sample should be set by the inelastic scattering length λ, which is supposed to increase with decreasing temperature T as an inverse power of T. The renormalization group analysis assumes that as the scattering length is increased, the flow rates $d\sigma_{xx} / d \log \lambda$ and $d \sigma_{xy} / d \log \lambda$ are completely determined by the values of σ_{xx} and σ_{xy} .

Though the precise form of the RG equations are not known, key features are common to all numerical and analytic work on the subject, and are in general agreement with experiments on quantum Hall systems. In one interesting approach, B. F. Dolan [2] employed a particular set of assumptions about the form of the RG equations, and obtained analytic results for the RG trajectories, which are illustrated in the left-hand panel of the figure below (reproduced from Checkelsky *et al.*) for the region $0 < \sigma_{xy} < 1$. The RG equations depend on σ_{xy} only modulo 1, so the trajectories are actually periodic in this variable. The RG flows have stable fixed points when σ_{xy} is equal to an integer and $\sigma_{xx} = 0$, which describes a simple insulator in the case $\sigma_{xy} = 0$, and integer quantized Hall states, otherwise. There is also a set of unstable fixed points where $\sigma_{xx} = 0.5$ and $\sigma_{xy} = 0.5 \bmod 1$. We shall be interested here in the fixed point at $\sigma_{xy} = 0.5$, whose outward flows connect to the stable fixed points at $\sigma_{xy} = 0$ and 1. These flows follow a semicircle, shown by the thick curve in Fig. 2, upon which the Hall resistivity $\rho_{yx} = 1$, while the longitudinal resistivity ρ_{xx} varies monotonically between infinity and zero. (Dolan's RG flows contain additional fixed points below this semicircle, which correspond to fractional quantized Hall states and transitions between them, but they will not concern us here.)

In the right-hand panel of the figure, taken from Checkelsky *et al*., we see experimental data taken over a range of temperatures between (roughly) 50 mK and 700 mK, at seven discrete gate voltages V_T. As seen in the figure, the seven data sets fall nicely on the RG trajectories predicted by Dolan's theory. The question is whether there is a deep meaning to this, or is this just a coincidence.

If one takes the RG predictions seriously, then in the limit of T=0, the seven trajectories should all flow to $\sigma_{xx} = 0$, with $\sigma_{xy} = 0$ or 1, depending on whether V_T is greater or less than a critical value V_C, whose value is approximately +4.2 V. Thus, there would be a sharp transition between the QAH state and a perfect insulator at this gate voltage at T=0. To the best of my knowledge, however, no physical mechanism has been proposed that would predict such a transition. The induced carrier density at this gate voltage would be a very small fraction of one carrier per unit cell of the two dimensional system.

The data points in the figure all have values of $\sigma_{xx} \geq 0.4$, so there is still a large extrapolation to the value of $\sigma_{xx} = 0$. However, data at the lowest temperatures fall close to the semicircle where $\sigma_{xy} = 1$. Thus, the Hall resistivity ρ_{yx} shows a well-defined plateau (slightly below the ideal quantized value) over a range $-2V \leq V_T \leq 6V$, even though the longitudinal resistivity is far from zero in this range. If the large values of σ_{xx} were due to thermally excited carriers, or some other "parallel conduction" mechanism, one might expect that the parallel carriers would make little or no contribution to σ_{xy}, since B=0, and it is not clear that such carriers would be strongly affected by the remnant magnetization. In this case however, one would have found a well-formed quantized plateau in σ_{xy} but not in ρ_{yx}.

Some details of the experimental system are as follows. The quoted composition parameters are x= 0.22 and y=0.8, and the ferromagnetic transition temperature was T_C = 45K. The film thickness was between two and three quintuple layers, and the electron mobility at T=80K was estimated as 270 $cm^2 V^{-1} s^{-1}$, which was smaller by a factor of 2.6 than the mobility of a similar film having the same value of y, but with Cr concentration x=0. Measurements in applied fields up to 14T, at an appropriate gate voltage, find values of σ_{xy} much closer to 1 and values of σ_{xx} much closer to 0 than the results at B=0.

In summary, these results are very provocative, but important questions remain to be answered. Theoretical work, as well as further experiments, would seem to be called for.

References.

[1]. C.-Z. Chang *et al*., "Experimental Observation of the Quantum Anomalous Hall Effect in a Magnetic Topological Insulator", Science **340**, 167 (2013).

[2] B. P. Dolan, "Modular invariance, universality and crossover in the quantum Hall effect", Nucl. Phys. B **554**, 487 (1999).