On Non-Fermi liquid phases due to Goldstone boson exchange

Criterion for stability of Goldstone Modes and Fermi Liquid behavior in a metal with broken symmetry

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Spontaneously breaking a continuous symmetry leads to massless Nambu-Goldstone modes (NGM)[1]. Examples are phonons (broken translation symmetry), spin-waves (broken spin rotation symmetry), or pions (broken chiral-flavor symmetries of massless quarks). The excitation of a Goldstone boson at longest wavelengths corresponds to a small variation of the order parameter within the degenerate manifold, leading to the vanishing excitation gap (masslessness). Such a variation of the order parameter is believed to be very weakly linked to other degrees of freedoms of the system. Thus, at long wavelength one expects NGM to decouple from other excitations, even if they are gapless themselves like particle-hole excitations in a metal. Fermionic degrees of freedom ψ and $\psi^\dagger$ are then expected to couple only to the gradient of the Goldstone bosons π:

$$\mathcal{H}_{grad} = \int dx \psi^\dagger \gamma_\mu \psi \partial^\mu \pi.$$

γ_μ is a problem-dependent coupling matrix and ∂^μ refers to the spatial derivative. In particle physics the vanishing coupling at small momenta is referred to as an *Adler zero*, where soft pions decouple from quarks[2]. The classic condensed matter example for an Adler zero is the gradient coupling of acoustic phonons to electrons. Another example is the gradient coupling between spin-waves and electrons in ordered antiferromagnets, as discussed by Schrieffer in the context of the cuprates[3].

Massless modes would have more dramatic impact in case of a contact or Yukawa coupling

$$\mathcal{H}_{contact} = \int dx \psi^\dagger \gamma \psi \pi,$$

like the coupling between fermions and gauge bosons or fermions and soft bosonic modes at a quantum critical point. The emergence of a contact coupling between NGM and electrons for a nematic state with elliptically distorted Fermi surface[4,5] was therefore surprising. It results for $d = 2$ and 3 in non-Fermi liquid behavior. In addition, NGM become over-damped, i.e. by slowly varying the order-parameter in the degenerate manifold one constantly excites particle-hole pairs.

Watanabe and Vishwanath derive a rather general criterion that allows to decide whether the interaction between NGM and fermions is via a gradient coupling or a contact coupling: Consider the generators Q of the broken symmetry and check whether they commute with the translation operator, i.e. the total momentum $\mathbf{P}$ (the generalization to discrete lattice translations of periodic crystals is straightforward). If $[Q, \mathbf{P}]_- = 0$, one obtains a gradient coupling, while a contact coupling occurs if the commutator is finite. If one breaks internal symmetries (e.g. spin rotation invariance in magnets without spin orbit coupling) the commutator vanishes and spin waves interact with electrons via a gradient coupling. The same is true for phonons where the translation symmetry is broken, i.e. $Q = \mathbf{P}$ and the commutator vanishes trivially. On the other hand, in case of the rotational symmetry breaking of the two-dimensional nematic state $Q = L_z$ with $[L_z, P_i] = i\epsilon_{ij} P_j \neq 0$, in full agreement with the contact coupling found in Ref.[4,5]. Using the above criterion the authors further propose a contact coupling to phonons for a crystal in an external magnetic field, an effect that should become significant if the magnetic length becomes comparable to the lattice constant of the crystal. Another proposal for rotational NGM coupled via a contact coupling to fermions was made for Rashba spin-orbit coupled itinerant ferromagnets[6,7].

Overdamping of Goldstone modes is known in the context of the A-phase of superfluid ^{3}He, where a rotation of the axis of the superfluid order parameter leads to massive reorganization of quasiparticles, i.e. damping[8]. What could be new in the nematic phase or the magnetic state with Rashba coupling is that fermions remain massless on extended parts of the Fermi surface. Whether the emerging non-Fermi liquid phase is indeed stable against the opening of a gap (see Ref.[9] for a discussion) and how the non-Fermi liquid physics feeds back on the assumed broken symmetry are exciting questions that result from these considerations. Whatever the outcome of these questions, the beauty of the work by Watanabe and Vishwanath are the simplicity and generality of their criterion $[Q, \mathbf{P}] \neq 0$ for NGM with contact coupling.

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