Topological quantum Lego

Imprint of topological degeneracy in quasi-one-dimensional fractional quantum Hall states

arXiv:1502.01665

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Some of the most beautiful discoveries of the last couple of years were made in the field of topological phases of matter. The simpler, non-interacting phases are included in the *periodic table of topological insulators*, and are distinguished by the appearance of the edge states protected by the bulk gap and the symmetry group of the underlying phase. While there are a lot of open problems about the non-interacting phases, I think it is fair to say that the most obvious and pressing concerns about them are resolved. The full classification of all the possible topological systems with all the possible kinds of symmetries might not yet exist; however, if one asks the right question, available tools guarantee finding the answer with a reasonable amount of effort. In particular, it is always easy to construct a topological model with a certain symmetry, and study all of its properties.

The situation is much less satisfactory, when it comes to the interacting and fractionalized systems. Historically, the first approach to these was to use trial wave functions, instrumental for the description of the fractional quantum Hall effect. This approach involves essentially guessing the correct solution from the start, and it requires therefore a great deal of ingenuity. Further, the trial wave functions do not always provide a recipe for constructing a Hamiltonian that produces them as a ground state. Often one can guess the kind of the Hamiltonian that would result in a desired behavior, and verify whether the guess is correct by using exact diagonalization or more advanced numerical techniques. An alternative approach is to construct exactly solvable models where all the terms in the Hamiltonian commute. These models were used to construct the most complicated topological phases, and in my personal understanding their limitation is that they rely on a very complicated and fine-tuned Hamiltonian in order to achieve what is needed. This means that perhaps the easiest way to construct topological phases through the exactly solvable models approach is to use a full-fledged quantum computer.

The work of Weizmann and Harvard team of researchers, highlighted above, uses the technique known as the 'wire construction', developed for fractional quantum Hall effect [1], and further extended to other phases in the last couple of years (see e.g. Ref. [2] for an application of the wire construction to the classification of topological phases). The main idea is to start with a simple initial Hamiltonian that cis constructed for a set of of parallel wires, each wire carrying a single electron mode and having a conventional dispersion relation. Then by using the standard bosonization technique we can switch to considering collective excitations in each wire. The next step is to construct the interactions and inter-wire hoppings in the wire array such that several conditions are fullfilled:

- All the extra hopping and interaction terms must commute with each other. This guarantees that these terms don't compete.
- All the degrees of freedom in the bulk must be gapped out by these extra terms.
- Some modes from the outermost wires may stay propagating. These are the topologically protected edge states.

This idea balances between the two approaches I mentioned above. Just like in the exactly solvable models the wire approach allows to retain an advantage of being able to start from a Hamiltonian, and taylor it to produce expected topological properties. This Hamiltonian is much simpler than that of exactly solvable models due to sacrificing of the requirement that all the terms in the Hamiltonian must commute: the kinetic energy of the wires does not need to commute with the inter-wire hopping and interaction terms. The price of the sacrifice is that now we are unable to figure out anymore the full excitation spectrum of the system, and are only able to predict correctly the ground state properties, the low energy spectrum, and sometimes the excitation gap.

The main physical phenomenon that the authors analyse is the $2\pi d$-periodic Josephson effect, the residue of the topological protection when one breaks the topological protection by reducing one of the linear dimensions of the system to be smaller than localization length, and additionally breaks translation invariance to remove the remaining accidental ground state degeneracy. In principle, this phenomenon could probably be predicted by generic topological arguments, entirely avoiding the need to perform a detailed analysis (the same holds also for many other topological phenomena). However the main reason why I consider the preprint important is the ability of the authors to carry out the analytical calculations beginning to end. Starting from the Hamiltonian, the authors are able to derive its topological properties, calculate the operators that switch the Hamiltonian between different ground states, and to address the splitting of the ground state degeneracy. There is also a nice feature that the authors do not advertise at all, namely the diagrammatic representation of the wire construction that allows to easily design and explain such topological phases. While it is merely a graphical representation of the underlying idea, I found it invaluable in developing an understanding of what exactly is happening.

As a final note, I would like to remark that the field of topological phases is largely theory-driven. The non-interacting topological systems proved to be a great experimental success, but there does not seem to exist a straightforward way to create fractional topological systems in a lab. I hope that the simple and robust methods used in the wire construction will allow to bridge the gap.

[1] C. L. Kane, Ranjan Mukhopadhyay, and T. C. Lubensky, Phys. Rev. Lett. **88**, 036401 (2002).

[2] Titus Neupert, Claudio Chamon, Christopher Mudry, and Ronny Thomale, Phys. Rev. B **90**, 205101 (2014).