Topological Mechanics

1) J. Paulose, A. Meunnissen, V. Vitelli, "Selective buckling via states of self-stress in topological metamaterials," arXiv:1502.03396

2) Z. Yang F. Gao, X. Shi, X. Lin, Z. Gao, Y. Chong, B. Zhang, "Topological Acoustics," Phys. Rev. Lett. **114**, 114301 (2015).

3) L.M. Nash, D. Kleckner, A. Read, V. Vitelli, A.M. Turner, W.T.M. Irvine, "Topological mechanics of gyroscopic metamaterials," arXiv:1504.03362.

Recommended with commentary by Chris Santangelo, UMass Amherst

Topological insulators have been an important area of quantum condensed-matter research for quite some time now, inspiring many a tired comparison between donuts and coffee mugs. Phenomenologically, a topological insulator is a material that is insulating in the bulk yet conducting on the surface. This insulating state is maintained by a wave function whose phase has a nontrivial winding number, inspiring the comment that such a material can exist only by virtue of its quantum mechanical nature [1]. A closer look at the mathematics, however, reveals that topological mechanisms also exist in the acoustic spectrum of a variety of classical systems from optics to flowing fluids to spring networks [2, 3]. Indeed, careful analysis of nonlinearities in the mechanical case has also uncovered new phenomena that do not arise in their topological quantum analogues, such as the directional propagation of solitons [4]; though in fairness, quantum systems exhibit a richer class of topological numbers than classical systems so far (though even this may be changing, see [5] and recommended papers 2-3).

For acoustic vibrations in spring lattices, the topological characterization arises for "isostatic" (or marginal) spring networks from the phonon band structure. An isostatic network is one for which there are precisely as many degrees of freedom as naive constraints. The phonon band structure is then encoded in a compatibility matrix, $\mathbf{Q}(\mathbf{k})$, which maps strains to stresses for deformations with wave vector $\mathbf{k}$ [3]; zero-energy phonons can only exist when $\det \mathbf{Q}(\mathbf{k}) = 0$. Since $\det \mathbf{Q}(\mathbf{k})$ is a *complex* polynomial, the existence of zeros is determined by how the phase winds around closed loops spanning the Brillouin zone [3] (in the study of polynomials, this is known as the argument principle). Ultimately, this provides a topological classification of rigid structures. As a consequence, however, a transition from one topological "phase" to another is characterized by the existence of zero-energy deformations.

The group of Vitelli in Leiden has done more than any other in understanding topological mechanisms in linkages. Most recently, they have begun exploration of how to use topology to *design* frameworks that act as mechanical "metamaterials," materials whose mesoscale structure lead to effective mechanical properties that are not found in naturally occurring materials (paper 1). They do this combining structures with different topological character to create localized regions of soft buckling and rigidity. And, of course, the topological character of this system ensures that this behavior is robust to imperfections in the structure or other types of damage. Whether this will turn out to be a useful way to engineer new materials remains unclear, but it is certainly an important step toward a Star Trek-ian dream of dialing up whatever properties you want in a new material. Another avenue to creating classical, topological states is explored by Yang *et al.* (paper 2). Their proposal involves creating a lattice rotation cylinders to drive fluid flow. The background circulation of the fluid modifies how acoustic waves propagate and, after a change of variables, has the mathematical form of the "zero field quantum hall" system [6]. A topological "quantum hall" phase of this sort is realized experimentally by Nash *et al.* (paper 3) in an "active" mechanical system constructed by gyroscopes.

Aside from fun with topology, there are lessons and potential useful tools emerging from the recent flurry of activity. On the theoretical side, it seems that not all rigid mechanical structures are rigid in the same way. On the practical side, there is the potential of using topology to design robust mechanical metamaterials. Beyond this, the apparent universality of the math suggests that there are likely to be many more topological systems in mechanics. One hopes that the nonlinearities arising in these systems generate a rich class of new phenomena beyond linear order that can be studied in table-top experiments.

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[1] Yet classical examples go back at least to 2008, for example see F. D. M. Haldane and S. Raghu, "Possible Realization of Directional Optical Waveguides in Photonic Crystals with Broken Time-Reversal Symmetry," Phys. Rev. Lett. **100**, 013904 (2008).

[2] V. Vitelli, Topological soft matter: Kagome lattices with a twist. Proc. Nat. Acad. Sci. (USA)

109, 12266-7 (2012).

[3] C. L. Kane and T. C. Lubensky, "Topological boundary modes in isostatic lattices," Nat. Phys. **10**, 39 (2013).

[4] B.G.-g. Chen, N. Upadhyaya, and V. Vitelli, "Nonlinear conduction via solitons in a topological mechanical insulator," Proc. Nat. Acad. Sci. (USA) **111**, 13004-9 (2014).

[5] Roman Süsstrunk and Sebastian D. Huber, "Observation of phononic helical edge states in a mechanical 'topological insulator'," arXiv:1503.06808.

[6] F. D. M. Haldane, "Model for a quantum Hall effect without Landau levels: condensed-matter realization of the parity anomaly," Phys. Rev. Lett. **61**, 2015 (1988).