Nonequilibrium Casimir forces: non-local, non-pairwise-additive, and not even extensive

Fluctuation-Induced Forces in Nonequilibrium Diffusive Dynamics
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Fluctuation–induced interactions are important in physics in many ways. They arise in any medium where fluctuations with long-range correlations are somehow modified by geometric constraints, e.g as imposed by inclusions and boundaries. While due to their generality, they have made their way into virtually all sub-fields of physics, they often bring new surprises when they are studied in new contexts. The original proposition of the effect—put forward by Hendrik Casimir in 1948—predicted the existence of a measurable force proportional to $\hbar$ between conducting plates; a macroscopic manifestation of quantum mechanics. In their most basic incarnation, the (van der Waals) forces are the very reason the liquid state of matter can exist, as an "incidental" finite domain in the phase diagram in between the solid phase (whose existence is guaranteed by the shot-range, or excluded-volume, repulsion between atoms and molecules), and the gas phase (that exists because interactions decay with distance). So, because of them we drink water from a glass rather than inhale it from a capsule!

Since these interactions are caused by fluctuations, they are proportional to the strength of what drives them; e.g. $k_{\mathrm{B}}T$ for thermal- and $\hbar$ for quantum-fluctuations. Due to the significance of the boundary conditions, they typically depend sensitively on the geometric characteristics of the system. Because of these two intrinsic features, fluctuation–induced interactions are non-local, i.e. stress in any given region is affected by fluctuations in far away places, and non-pairwise-additive, i.e. interactions among three objects are not the same as the sum of three pairwise components.

Equilibrium fluctuation–induced forces are known to be extensive; the interaction between two flat plates is proportional to their area, provided the distance between them is smaller than their lateral size. When this property, which is a consequence of the lateral translational symmetry in the geometry, is combined with the above two, we have enough information to deduce the form of the force between two parallel plates of side L that are separated by a distance H in d-dimensions; $F \sim k_{\mathrm{B}}TL^{d-1}/H^d$ for thermal- and $F \sim \hbar c L^{d-1}/H^{d+1}$ for quantum-fluctuations. This makes sense if we think about the boundary conditions as "filters" that select certain modes in the fluctuations and forbid others when counting the modes that contribute to the zero-point energy, in a manner that is dictated by the boundary conditions, and hence the geometry. This suggests that when we introduce time-dependent boundary conditions, say by moving conducting plates in the quantum vacuum, we will enforce a dynamical transition in the system that might often require it to go from one energy level to a lower one in a short time, and hence cause vacuum to emit photons to be able to achieve that. This transient dynamics will lead to a frictional fluctuation–induced force that takes energy from the mechanical source and converts in into radiation via the quantum vacuum; a phenomenon that is inherently related to Hawking radiation and the Unruh effect.

Nonequilibrium stationary states offer a promising new direction for fluctuation–induced forces, as they provide a generic route to the emergence of fluctuations with long-range correlations in cases where the equilibrium counterparts exhibit short-range correlations [1–4]. In a recent paper, Aminov et al [5] have used the continuum limit of the so-called symmetric simple exclusion process in a system maintained between two particle reservoirs with different concentrations (C_l and C_r) to study the forces induced by nonequilibrium concentration fluctuations. For two walls of side L positioned along the gradient in a pipe geometry of square cross section of size H (see Fig. 1), they find a force

$$F \sim \frac{k_{\mathrm{B}}T}{H} \cdot (C_r - C_l)^2 a^{2d} \sim \frac{k_{\mathrm{B}}T}{H} \cdot L^2 |\nabla C|^2 a^{2d}, \tag{1}$$

where a is a short distance cutoff inherited from the underlying lattice (and hence a^d is the excluded volume of each particle). The prefactor in Eq. (1) changes between two finite numbers when crossing over from $H \ll L$ to $H \gg L$; an extraordinary feature.

Surprisingly, the force between the two segments depends in a non-extensive manner on the length scale L along the gradient. Moreover, as Eq. (1) demonstrates the way the force depends on L is determined by what is kept constant when L is changed: it is independent of L if $C_r - C_l$ is kept constant, whereas it is proportional to L^2 if ∇C is kept constant. An immediate consequence of this is that the nonequilibrium force for objects with small curvature

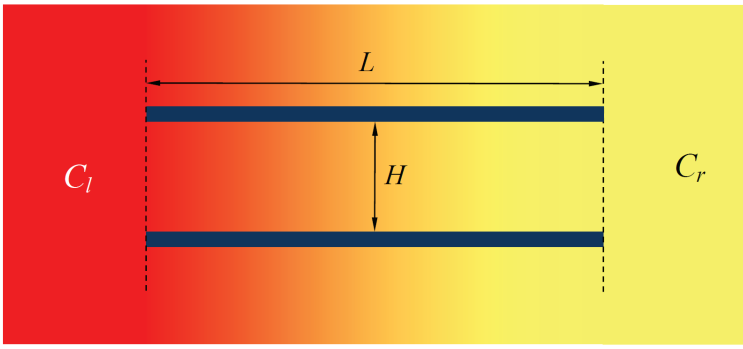

FIG. 1 Schematics of the system with the two reservoirs with different concentrations and the boundaries that do not let the particles pass through.

(relative to the relevant distance) cannot be approximated using the celebrated Derjaguin approximation scheme [6], which is based on treating the system as locally flat segments and adding up their piecewise-extensive contributions. Consequently, how non-ideal geometric features such as curvature will modify Eq. (1) is far from trivial.

How to treat cut-off dependent terms (to avoid UV divergencies) is in general a delicate issue in the calculation of the fluctuation–induced forces, and this particular study has its own share of this complication due to the interplay between the lattice structure, the continuum limit, and the geometry of the system. Aminov et al have shown that a subtle scheme of ordered limits will help remove the cutoff dependent terms and yield a result that depends only on the excluded volume of the particles, as seen in Eq. (1). This manifestly shows that the nonequilibrium fluctuation–induced force is a result of the particle-particle interaction, and is absent in nonequilibrium steady states with non-interacting particles.

Much like the original Casimir effect teaches us something about quantum mechanics from measurements of mechanical forces between macroscopic objects, this work—and others that will no doubt follow—will provide us with an opportunity to design a new generation of experiments through which we can learn about nonequilibrium steady states. For example, the work is based on a number of assumptions that has been used as input to the model, such as a local fluctuation–dissipation theorem that relates concentration fluctuations to the compressibility in a local manner and a local dependence of pressure on the concentration. Experimental probe of the force will help us verify or test these assumptions.

Casimir effect seems to continue to play its role in surprising and enthusing us at the same time. Remember that next time you drink a glass of water!

References

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