Getting a grip on quantum criticality in metals

i. Ising nematic quantum critical point in a metal: a Monte Carlo study
Authors: Yoni Schattner, Samuel Lederer, Steven A. Kivelson, Erez Berg
arXiv:1511.03282.

ii. The nature of effective interaction in cuprate superconductors: a sign-problem-free quantum Monte-Carlo study
Authors: Zi-Xiang Li, Fa Wang, Hong Yao, Dung-Hai Lee
arXiv:1512.04541

iii. Competing Orders in a Nearly Antiferromagnetic Metal
Authors: Yoni Schattner, Max H. Gerlach, Simon *Trebst*, Erez Berg,
arXiv:1512.07257.

Recommended with a Commentary by Jörg Schmalian, Karlsruhe Institute of Technology

A Fermi surface is a rather complicated object. While the low-energy behavior of bosons is determined by isolated points in momentum space, dense Fermi systems are governed by a manifold of codimension one. For generic systems it was shown by Landau in his theory of Fermi liquids how to deal with this issue. For metals near a quantum critical point, singularities in the fermion-fermion scattering amplitude are expected that go beyond the ones captured by Landau's theory. Thus, Fermi liquid theory, that has been the foundation of the theory of metals in two and three dimensions, may not apply. Despite important steps towards a solution, metallic quantum criticality remains one of the major unsolved problem in condensed matter physics. It is believed to be of importance for correlated oxides (e.g. cuprate high-T_c superconductors, ruthenates), the iron based superconductors, and heavy fermion systems.

A popular approach to metallic quantum criticality is based on a low-energy theory with a collective bosonic order-parameter φ that interacts with otherwise free fermions via the coupling

$$H_{\rm int} = g \int \varphi \psi^\dagger O \psi. \tag{1}$$

g determines the strength of the interaction and $\psi^\dagger O \psi$ is some fermionic bilinear, with O standing for the vector of the Pauli matrices (for magnetic critical points) or acts on the

coordinates (in the case of a nematic instability where rotational symmetries are broken). With no obvious small parameter in sight, analytical techniques have been based on the expansion in the inverse number of fermionic flavors ($1/N_F$) or bosonic flavors ($1/N_B$). While the $1/N_F$-expansion was shown to be significantly more complex that previously believed[1], the $1/N_B$-expansion seems to be limited to high, possibly intermediate energies[2].

Important progress was recently made in the above manuscripts by formulating appropriate lattice versions of two-dimensional fermion-boson models with interactions of the type Eq.(1) and devising Quantum Monte Carlo (QMC) simulations that are free of the unsavory *minus-sign-problem*. Schattner *et al.* [*i*] consider an Ising nematic quantum critical point and avoid the sign problem because the operator O is trivial in spin space. Li *et al.*[*ii*] and Schattner *et al.*[*iii*] consider antiferromagnetic order and fluctuations. Following Ref.[3] they analyze a two-band version of the theory that also avoids negative probability densities. Note, that similar investigations, with specific focus on the physics of the iron based superconductors, were recently performed by Li, Wang, Yao, and Lee[5] and Dumitrescu, Serbyn, Scalettar, and Vishwanath[6]. In particular, the latter analysis is performed for a purely electronic model that naturally leads to strong nematic fluctuations.

Let us first discuss the paper by Schattner *et al.* [*i*] who study a particularly well defined problem: an Ising-nematic transition where a C_4-symmetric system spontaneously breaks this symmetry down to C_2. This paper makes the most direct statement about the universality near a metallic quantum critical point, which, in the language of Fermi-liquid theory, is a $l = 2$ Pomeranchuk instability. It studies a transverse-field Ising model (representing the nematic order parameter φ) coupled to fermions according to Eq.1. First of all, the authors do find a metallic quantum critical point! The coupling to fermions changes the universality class from $2 + 1$-dimensional Ising, yet it does not lead to a first order transition or another instability (at least for the finite temperatures studied). The authors see at best very modest renormalizations of the fermionic degrees of freedom with small or zero anomalous dimension (the latter corresponds to an ordinary Fermi liquid). In addition, they analyze the momentum, frequency, temperature, and transverse-field dependence of the boson two-point function and obtain at low energies a behavior well described by

$$D\left(\mathbf{q},\omega_n\right) = \frac{A}{T + b\left(h - h_c\right) + \kappa\mathbf{q}^2 + c\left|\omega_n\right|}, \tag{2}$$

where h is the transverse field and h_c its value at the quantum critical point (A, b, κ, and

c are numerical constants). The static form of this propagator is consistent with mean field exponents $\nu = 1/2$, $\gamma = 1$, and $\eta = 0$, i.e. qualitatively different from the transverse-field Ising model without the coupling to fermions. Except for hard to identify logarithmic corrections the static behavior is fully consistent with a simple one-loop analysis. However, the frequency dependence yields $z = 2$ ($\mathbf{q}^2 \sim |\omega_n|$), instead of $z = 3$, a behavior expected by particle-hole excitations, reflecting Landau-damping. Even though it was shown in Ref.[1] that the $1/N_F$ expansion is much more complex for two-dimensional systems than the simple loop-expansion that yields $z = 3$, the value of z seems to be among the few properties not affected by those complications (see however for[4] for the possibility of logarithmic corrections to $z = 3$). While $z = 2$ was in fact obtained in the large-N_B limit[2], the concomitant large anomalous dimension of fermions of Ref.[2] does not seem to occur in the simulation. Thus, none of the known analytic approaches seems to naturally explain the findings by Shattner *et al.* [*i*]. Even if Eq.2 is only valid at intermediate energies, the quite convincing scaling behavior over more than a decade suggests that the system is affected over a wide energy regime by a new fixed point. To get a better understanding of this new fixed point is a sharply defined challenge to the community.

The manuscripts by Li *et al.*[*ii*] and Schattner *et al.*[*iii*] study antiferromagnetic fluctuations, with a range of highly interesting results. Both find a strong tendency towards d-wave superconductivity, putting on firm ground what was obtained earlier in other, approximate approaches of the same model. In addition, both demonstrate that *d-wave pairing is most pronounced near the antiferromagnetic quantum critical point* and show that pronounced short range charge density-wave correlations, yet no long range charge density wave order emerges. Li *et al.*[*ii*] also consider nematic fluctuations (i.e. allow for two different kinds of collective bosons) and demonstrate that *spin-fluctuation induced d-wave pairing is boosted by nematic fluctuations*. Overall, these results seem to capture important aspects of the physics of iron-based superconductors (see also[5,6]). On the other hand, it is less clear whether the physics of the pseudogap, a hallmark of the cuprate superconductors that is at best weakly visible in these simulations, is fully captured by models of free fermions coupled to collective bosonic modes.

The above manuscripts have in common that they study lattice versions of models widely discussed as low energy theories of metallic quantum critical points. The models are devised to avoid the minus sign problem of Monte Carlo simulations, allowing the study of inter-

mediate and low energy behavior for reasonably large systems (up to 24×24 lattice sites). This opens the door to study universal behavior of metallic quantum critical points reliably. While it is unclear at the moment whether there exists a realistic (e.g. Hubbard like) model of strongly interacting fermions with a quantum critical point, that is asymptotically equivalent to free fermions coupled to collective bosons, a study of such effective theories is sufficiently interesting on its own right. The insights obtained in these preprints clearly help us to get a firmer grip on quantum criticality in metals or the emergence of secondary order near quantum critical points.

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