Topological magnetoelectric effect *versus* quantum Faraday effect

- **Observation of topological Faraday and Kerr rotations in quantum anomalous Hall state by terahertz magneto-optics**

  K.N. Okada, Y. Takahashi, M. Mogi, R. Yoshimi, A. Tsukazaki, K.S. Takahashi, N. Ogawa, M. Kawasaki, and Y. Tokura, arXiv:1603.02113

- **Quantized Faraday and Kerr rotation and axion electrodynamics of the surface states of three-dimensional topological insulators**

  L. Wu, M. Salehi, N. Koirala, J. Moon, S. Oh, and N.P. Armitage, arXiv:1603.04317

- **Observation of the universal magnetoelectric effect in a 3D topological insulator**

  V. Dziom, A. Shuvaev, A. Pimenov, G.V. Astakhov, C. Ames, K. Bendias, J. Böttcher, G. Tkachov, E.M. Hankiewicz, C. Brüne, H. Buhmann, and L.W. Molenkamp, arXiv:1603.05482

Recommended with a commentary by Carlo Beenakker, Leiden University

This March three independent reports appeared of a quantized magneto-optical effect in a topological insulator, thin films of HgTe, Bi_2Se_3, and Cr-doped $(Bi,Sb)_2Te_3$. Time-reversal symmetry is broken either externally by a perpendicular magnetic field, or internally by the magnetic Cr impurities. It was found that the polarization of a THz electromagnetic plane wave transmitted through the film at normal incidence is rotated by an angle $\theta = \arctan\alpha$ set by the fine structure constant $\alpha = e^2/4\pi\epsilon_0\hbar c \approx 1/137$.

This quantized Faraday rotation originates from the quantization of the optical Hall conductivity $\sigma_{xy} \to \nu e^2/h$ ($\nu = \pm 1, \pm 2, \ldots$), at vanishing longitudinal conductivity $\sigma_{xx} \to 0$. For the thin film in the experiments (thickness $d \simeq 10\,\text{nm}$ much smaller than wavelength $\lambda \simeq 100\,\mu\text{m}$) the rotation angle and conductivity are related by

$$\tan\theta = \frac{\sigma_{xy} Z_0}{1 + n_{\text{sub}} + \sigma_{xx} Z_0} \to \frac{2\alpha\nu}{1 + n_{\text{sub}}}, \tag{1}$$

with $Z_0 = 2\alpha h/e^2$ the vacuum impedance and $n_{\text{sub}} \geq 1$ the dielectric constant of the substrate. This is the *quantum Faraday effect* of Volkov & Mikhailov [1]. Previous experiments in 2D systems (GaAs [2] and graphene [3]) showed plateaus in the rotation angle as a function of magnetic field, but the accuracy of the quantization was poor (even after accounting for the substrate correction).

The topological insulators are 3D systems that show a quantum Hall effect because of the 2D conducting surface. In a slab geometry the top and bottom surfaces each contribute $\frac{1}{2} \times e^2/h$ to the Hall conductivity, for a total $\nu = 1$. By measuring the rotation angle in reflection (Kerr rotation) as well as in transmission, the substrate contribution could be eliminated and a nicely quantized plateau was obtained: Wu et al. quote an accuracy in the measured fine structure constant of 0.5% and suggest that with further refinement the quantum Faraday effect could have applications in metrology.

This is an impressive achievement, but the reader might ask why I chose these papers for discussion in the Condensed Matter Journal Club — *Is there anything to discuss?* I invite a discussion on whether the observed quantum Faraday effect (QFE) demonstrates the topological magnetoelectric effect (TME).

The topological magnetoelectric effect (Qi, Hughes & Zhang [4]; with this discussion [5]) is the production of a quantized electrical polarization

$$\boldsymbol{P} = \frac{\theta}{2\pi}\frac{e^2}{h}\boldsymbol{B} \tag{2}$$

in response to a magnetic field, and its dual, a quantized magnetization in response to an electric field. Both responses follow upon addition of an $\boldsymbol{E}\cdot\boldsymbol{B}$ term

$$\mathcal{L}_\theta = \frac{\theta}{2\pi}\frac{e^2}{h}\boldsymbol{E}\cdot\boldsymbol{B} \tag{3}$$

to the usual Maxwell Lagrangian, providing a condensed matter realization of the axion electrodynamics from particle physics (Wilczek [6]). The axion angle $\theta = 0$ in an ordinary insulator while a topological insulator has $\theta = \pi$, both angles modulo 2π. In a microscopic description, θ represents a contribution to the magnetoelectric polarizability from extended orbitals (Essin, Moore, & Vanderbilt [7]).

The mechanism by which a perpendicular magnetic field polarizes a slab of topological insulator is illustrated in Fig. 1a: One needs to bias the slab such that the top and bottom surfaces, at $z = \pm d/2$, have *opposite* Hall conductivities $\pm e^2/2h$. (Methods to achieve this are discussed by Morimoto et al. [8] and Wang et al. [9]) The magnetic field then produces an opposite charge density $\pm Be^2/2h$ ($\pm e/2$ per flux quantum) on the two surfaces. The resulting dipole moment $d \times Be^2/2h$ (per unit surface area) corresponds to a bulk electrical polarization (per unit volume) of $Be^2/2h$, in accord with Eq. (2) for $\theta = \pi$. This is the TME. No Faraday rotation is produced, because top and bottom surfaces rotate the electromagnetic field in opposite directions. To observe the QFE one needs the setup in Fig. 1b, with the *same* Hall conductivity $e^2/2h$ on top and bottom surface, so that the rotation angles add rather than cancel, but then a magnetic field cannot produce a charge imbalance between the surfaces so there's no TME.

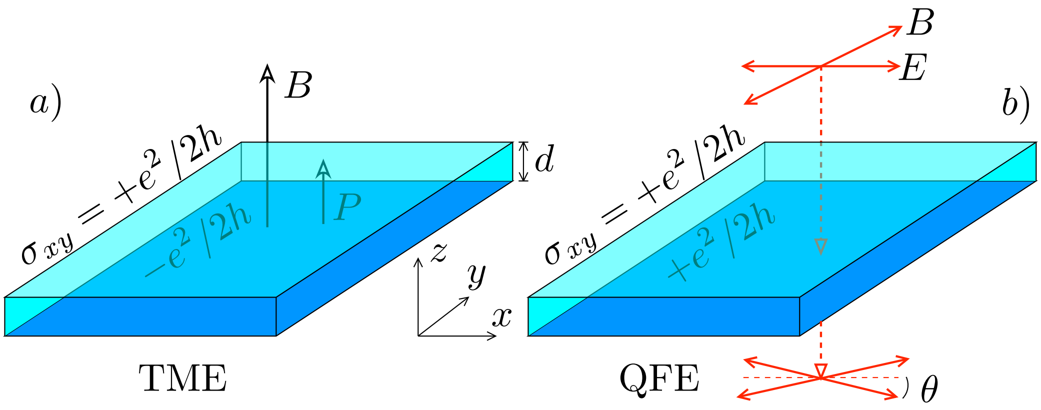

Figure 1: Topological insulator films with opposite (panel a) or the same (panel b) Hall conductivity σ_{xy} on top and bottom surfaces. Panel a) allows for a measurement of the topological magnetoelectric effect (TME, an electrical polarization P in response to a magnetic field B), while panel b) allows for a measurement of the quantum Faraday effect (QFE, a quantized rotation θ of the polarization angle of a transmitted electromagnetic wave). The TME vanishes in panel b) while the QFE vanishes in panel a), in this sense the TME and QFE are mutually exclusive.

The TME is a manifestation of axion electrodynamics, so one might ask (with Tse & MacDonald [10]) whether the topologically nontrivial axion angle θ could be deduced directly from the QFE. One would then need to independently have access to the optical rotation angles of the top and bottom surfaces — otherwise one can only obtain 2θ without the possibility to tell whether θ equals 0 or π modulo 2π. Such independent measurements are possible by the combination of Faraday and Kerr rotations (Maciejko et al. [11] describe how), but not for films much thinner than the wavelength. I am inclined to conclude that for such thin films there is no fully optical route to the demonstration of the topological magnetoelectric effect nor of axion electrodynamics.

I have benefited from correspondence with Akira Furusaki, Ewelina Hankiewicz, Allan MacDonald, and Joel Moore. Any misinterpretations are entirely my own, and I welcome further discussion.

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