Superconducting instabilities of quantum critical metals

Are non-Fermi-liquids stable to Cooper pairing?

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Recommendation and Commentary by Leon Balents, KITP, UCSB

For many years it has been apparent empirically that superconductivity often appears at low temperature to "cover" putative quantum critical points in metallic systems. This is best documented probably in the heavy fermion materials, where many quantum critical points associated with magnetic order have been identified, but similar phenomenology is observed also in several transition metal oxides and some organics. Another common feature of these quantum critical systems is systematic deviations from the naïve expectations of Fermi liquid theory in the metallic state above the superconducting critical temperature. The appearance of such non-Fermi liquid properties at quantum critical points is expected and has been discussed theoretically since the 1970s, though the theory itself is challenging and has undergone several revisions since early work of Hertz.

The basic physics which makes quantum critical points special is the presence not only of low energy electron and hole excitations, which are present in any metal, but also of "soft modes" associated with the order parameter of the phase transition. The additional collective low energy excitations have two effects. They open a new channel for electrons to lose energy and momentum by emitting a collective mode. They also mediate an effective interaction between electrons, just as phonons do in conventional superconductors. The former effect, if there is enough phase space, may lead to scattering that exceeds the Fermi liquid rate $1/\tau_{FL} \sim \epsilon^2$, where ϵ measures the energy of quasiparticles, or their temperature, or both. If the scattering rate becomes comparable to the energy of the excitation itself, then quasiparticles cease to be well defined. Some basic questions arise: How does one describe such a metal without true quasiparticles? And how do you think of superconductivity, if not as binding of quasiparticles into pairs? Does the strong quasiparticle scattering inhibit superconductivity?

An important distinction to be made is whether the incipient order breaks translational symmetry of the metallic state. If so, the soft order parameter modes carry a non-zero momentum Q, and scatter predominantly electrons only near "hot spots" separated by this momentum transfer. If not, a zero momentum order parameter scatters all the electrons on the Fermi surface, and does so only by small angle scattering. It happens that similar physics can occur in other situations, away from quantum phase transitions, in which some other long wavelength low energy excitations play the role of the soft modes. In particular, this includes the problem of fermions interacting with a massless gauge field, which arises in the theory of the half filled Landau level in the quantum Hall effect, and in certain quantum spin liquid states of frustrated magnets.

Together with Q=0 quantum critical points in metals, these cases are considered by Metlitski *et al.* Their starting point is a recently developed "two patch" model for non-Fermi liquid effects, in which each pair of antipodal regions on the Fermi surface is considered to be independent from other regions[1][2]. This was argued to capture the leading effects of the interaction with the collective mode. Within the two patch model, a double expansion using both large N and ϵ-expansion techniques (see the paper for an explanation of N and ϵ) enables a controlled calculation of non-Fermi liquid properties[3]. (see http://www.condmatjournalclub.org/?p=1195 for a prior discussion on this site).

However, superconductivity arises from "Cooper" processes that take a zero momentum pair of electrons from one pair of patches to another, and so is outside the two patch theory. In the Metlitski *et al* paper, the authors introduce coupling between different patches using a renormalization group scheme introduced by Dam Son in the context of quark matter. The rough idea is to subdivide patches into sets of smaller ones as the system is rescaled, which leads to Cooper interactions between nearby regions of the Fermi surface. These then in turn can grow, if attractive, to generate superconductivity.

Despite the authors' dubious choice to phrase the title as a question, they do come to some conclusions in this way. They argue that the electronic nematic quantum critical point is indeed *unstable* to superconductivity, and that the pairing scale for this superconductivity is "large". Conversely, they find that the problem of fermions interacting with a gauge field is locally stable to pairing. It instead exhibits a phase transition to a paired state if an attractive interaction exceeds some non-zero threshold. If correct, these results are important cornerstones in our understanding of non-Fermi liquid metallic states.

Some questions arise due to the reliance upon a double expansion – two wrongs do not necessarily make a right, though such sins are common amongst theorists. An interesting feature of the results is that in fact in the nematic case the pairing is so strong that the non-Fermi liquid regime is preempted by the superconducting transition. This may be an artifact of the double expansion. If it is in fact a real feature, it would signal a challenge for theory in explaining robust non-Fermi liquid behavior above superconducting states. Indeed Metlitski *et al* remark that their results for pairing can be obtained by a much more conventional Eliashberg-like calculation, which largely ignores non-Fermi liquid physics. We may also wonder whether the separation into patches itself is good beyond the expansion.

Regardless, the paper provides plenty of food for thought. It will be interesting to see how it ultimately impacts our interpretation of quantum criticality, non-Fermi liquid behavior, and unconventional superconductivity in experiment.

References

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