**Observation of the chiral anomaly in solids.**

1. Signature of the chiral anomaly in a Dirac semimetal: a current plume steered by a magnetic field
   Authors: Jun Xiong, Satya K. Kushwaha, Tian Liang, Jason W. Krizan, Wudi Wang, R. J. Cava, N. P. Ong
   ArXiv:**1503**.08179
   and several other papers. (see ref 11-14 in commentary)

2. Chiral anomaly and classical negative magnetoresistance of Weyl metals
   Authors: D. T. Son and B. Z. Spivak
   Phys. Rev. B **88**, 104412 – Published 13 September 2013

**Recommended with a commentary by Patrick Lee, MIT**

Chiral anomaly, commonly known as the Adler-Bell-Jackiw anomaly after the discovery made in 1969, is an important concept in field theory which plays a key role in the standard model of particle physics. As pointed out by Hermann Weyl in 1929, the four component massless Dirac equation in 3+1 dimensions can be separated into two two-component equations

$$i\,\frac{\partial\psi}{\partial t} = \pm\, c\,\vec{p}.\vec{\sigma}\,\psi \qquad (1)$$

where $\vec{\sigma}$ are the Pauli matrices Eq (1) describes particles with a definite chirality $\vec{\sigma}.\hat{p}$, which we now refer to as Weyl fermions. According to the classical equation of motion, the number of fermions with plus or minus chirality is separately conserved. The statement of chiral anomaly is that in the quantum field theory of Weyl fermions coupled to electromagnetic gauge field, $N_\chi$, the number of fermion carrying chirality of sign $\chi$ is no longer conserved, but obeys the anomaly equation:

$$\frac{dN_\chi}{dt} = \frac{\chi e^2}{4\pi^2\hbar^2 c}\,(\vec{E}.\vec{B}) \qquad (2)$$

The text book explanation of this apparent puzzle is that quantum field theory requires a consistent short distance regularization, and the separate conservation of chiral fermion number is inconsistent with gauge invariance.

In a remarkable paper back in 1983, Nielsen and Ninomiya [1] provided a more physical explanation of the chiral anomaly in a language which is familiar to condensed matter physicists. They started with a one dimensional example of a partially filled tight binding band on a lattice. At the chemical potential we have left and right movers shown in Fig 1 which would appear to be separately conserved if there is no scattering between them. However, a solid state physicist will know that these bands are connected far below the Fermi surface and in the presence of an electric field $\vec{E}$, for positively charged particles the momentum state flows according to the simple equation $\dot{k}=eE$. Thus in the presence of an electric field, charge flows from left to right as shown in Fig 1, and the number of right movers obey

$$\dot{N}_R = \frac{e}{2\pi}\,E \qquad (3)$$

Eq (3) is the anomaly equation in 1+1 dimension.

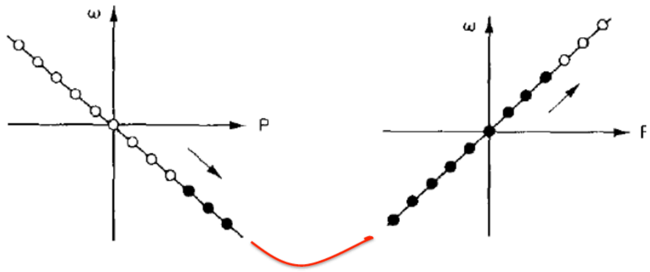


Fig 1. The left and right movers in a one dimensional band are connected far below the Fermi level, allowing charge flow between them. Adapted from Ref. 1.

Nielsen and Ninomiya offered a similar explanation for 3+1 dimension. Consider a band structure which supports two Weyl nodes with opposite chirality separated in momentum space and apply a magnetic field in the $z$ direction. A simple extension of the familiar 2D graphene case gives eigenvalues of the form

$$E_n(p_z) = \pm v\,(2ne\hbar B/c + p_z^2)^{1/2}. \tag{4}$$

The dispersion for one of the nodes is shown in Fig 2. As is familiar from graphene the special feature of Landau levels for 2D Dirac band is the existence of a zero energy mode. This mode extends to a one dimension band in the $p_z$ direction for the 3D Weyl fermion. This band connects the two Weyl nodes just as in the 1D case shown in Fig 1. and serves as the conduit for charge pumping between the two nodes in the presence of an electric field parallel to B. The flow of charge is given by the analog of Eq(3), except that we need to include the degeneracy of the zero mode which is $eB/h$ per area normal to B. The right hand side of Eq (3) is then proportional to $E_zB = \vec{E}.\vec{B}$ and yields exactly Eq(2).

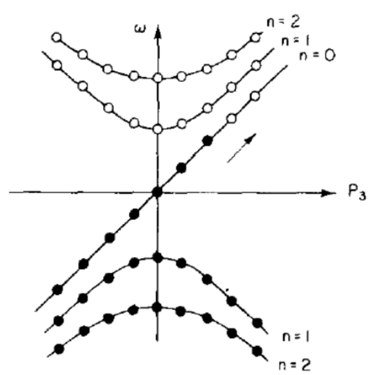


Fig 2. The spectrum vs the z component of the momentum for a 3D Weyl fermion in a magnetic field. (From ref.1)

Thus the message is that for a solid state physicist, the anomaly equation is perfectly normal, because on a lattice a consistent short distance cut-off is built in. Nevertheless the notion of charge pumping between Weyl nodes led Nielsen and Ninomiya to predict an enhanced magneto-conductance which depends on $\vec{B}.\hat{E}$. This prediction remained untested for over 30 years, because there were no known examples of Weyl nodes in solids. The recent explosion of interest in topological band structures and related concepts such as Berry's curvature has changed all that. The interest in Weyl semimetals began with the theoretical proposal of Wan et al [2] and by now several examples have been predicted theoretically and observed experimental. For an excellent review, see the commentary by Ashvin Vishwanath in the February 2015 issue of The Journal Club. Briefly, the four component Dirac equation arises naturally in solids when two doubly degenerate levels (due to spin for instance) cross. However, the crossing will generically be gapped unless protected by crystalline symmetry. The understanding of how this symmetry protection works [3] led to the prediction [4,5] and observation of $Na_3Bi$ and $Cd_3Se_2$ [6-10] as gapless 3D Dirac semimetals. Starting with gapless Dirac nodes, the breaking of either time reversal or inversion symmetry will split each Dirac node to form a pair of Weyl fermions.

Meanwhile the solid state physics perspective of the chiral anomaly has been further advanced by the paper of Son and Spivak. They show that the anomaly equation and enhanced magneto-conductance can also be understood in the semi-classical limit, using the conventional Boltzmann equation which describes the distribution of carriers near the Fermi surface, which in a semimetal may lie quite a bit above or below the Weyl nodes. Landau quantization is not required. The key ingredient is that the Weyl nodes are monopole sources of Berry's curvature. [2] Once the Weyl nodes are separated, these Berry curvatures give an important contribution to the anomalous velocity in a semi-classical description and magically reproduces the anomaly equation. Son and Spivak provided a formula for the anomaly contribution to conductivity which they showed to be proportional to $(\vec{B}.\hat{E})^2$ in the semiclassical limit, ie when many Landau levels are occupied. Together with the original Nielsen and Ninomiya work, all mysteries have been taken out of the notion of chiral anomaly for solid state physicists, and what remains is experimental confirmation.

In the past few months, several papers appeared in rapid succession reporting the discovery of anomaly related phenomena including unusual negative magnetoresistance in Weyl semimetals. I recommend the paper by Xiong et al because it is a summary of a talk at the APS March meeting and contains a comprehensible introduction to the subject and context. It also contains extensive data on the anisotropy of the magnetoresistance, which the authors describe as an enhanced conductivity plume steered by the magnetic field. Note that unlike conventional magnetoresistance, this plume is insensitive to lattice direction, but depends only on the angle between the current and the applied field.

So far this unusual negative magetoresistance has been reported in four materials by five research groups [11-14 and Xiong et al] and can be divided into two classes. In the first case (TaAs) the Weyl nodes are created by the intrinsic breaking of inversion symmetry of the crystal structure [12,13]. Here the band structure is quite complicated, with 24 Weyl nodes. In the second case the Weyl nodes are created by breaking time reversal symmetry using the magnetic field itself. Example include $ZrTe_5$ [11], $Na_3Bi$ [Xiong et al] and $Cd_3As_2$. [14] Here we start with 2 Dirac nodes which are split into 4 Weyl nodes. While qualitatively the locking of the enhanced current to the magnetic field direction is well documented, Xiong et al pointed out that the angular width of the plume is narrower than expected. Thus a quantitative understanding of the data is still lacking. It should be noted that all these materials exhibit a remarkable large linear magnetoresistance for current perpendicular to the magnetic field which is not understood. A quantitative explanation of the full magneto-transport awaits further theoretical development, but that is a story for another day.

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