No Go Theorems in Interacting Fermions

1. Limits on dynamically generated spin-orbit coupling: Absence of $\ell = 1$ Pomeranchuk instabilities in metals.
arXiv:1611.01442
Authors: Egor I. Kiselev, Mathias S. Scheurer, Peter Wölfle, and Jörg Schmalian

2. Conservation and persistence of spin currents and their relation to the Lieb-Schulz-Mattis twist operators.
Phys. Rev. B 80, 012401 2009.
Authors: N. Bray-Ali and Z. Nussinov

Recommended with a Commentary by Chandra Varma, University of California, Riverside.

In a Galilean invariant system, for which the Landau theory of Fermi-liquids was originally designed, the (velocity-independent) interactions are the same in the moving frame as in the stationary frame. Therefore interactions do not renormalize the current operator. It then follows that, among other things, instability due to interactions to a state with uniform current is not possible. If it were not forbidden, it would amount to a dipolar deformation of the Fermi-surface, which would be the $\ell = 1$ spin-symmetric Pomeranchuk instability. Such a state occurring spontaneously is also forbidden more generally than Galilean invariance by gauge-invariance or the proposition that a photon cannot acquire mass, sometimes called Elitzur's theorem.

Such a state was proposed by Heisenberg as representing the superconducting state[1]. In response F. Bloch is said to have stated, but never published, that such a state is impossible. The proof of this Bloch's theorem, for a Hamiltonian with any one-particle potential (for example a periodic lattice) and multi-particle interactions which are purely density dependent, was provided by D. Bohm in an elegant and simple variational calculation [2]. The argument is simply that for any variational wave-function carrying uniform current, one can construct another with energy decreasing linearly with lower current carried. Consistent with the Bloch-Bohm theorem, supercurrents only flow in metastable states or in response to variations in magnetic field as required by the Meissner-Ochsenfeld effect in superconductors.

The argument against uniform current-carrying states can also be made from conservation laws and Landau theory for velocity independent Hamiltonians which are not Galilean invariant. There is no contribution using the incoherent parts of the single-particle Green's function for conserved quantities such as density and spin-density. This together with the associated continuity equations or Ward identities allows no renormalization for current-current correlations either of spin or charge. This is shown quite simply by Kiselev et al. in the first paper noted above.

None of the above implies that current carrying states at $q = 0$ is disallowed in a lattice with a basis. Such states were proposed and are in fact have been observed in cuprates [3] and in an iridate compound [4].

The issue of spin-currents is more fraught. In problems with spin-orbit interaction, which is where most of the interest lies, spin is itself not conserved and so there is no continuity equation. But let us first consider the case that the total local spin and all its components are conserved and there is a spin continuity equation, for example in a problem which is again with only density dependent interactions. Then the second paper noted above has generalized Bloch's argument to show that no equilibrium state carrying uniform spin-current is possible. This has also been derived using a Ward identity in the first paper above. So Pomeranchuk instability is also impossible in the $\ell = 1$ antisymmetric spin-channel in the absence of a spin-orbit interactions. Again more complicated spin-current patterns on lattices are not disallowed. These are described by Kiselev et al.

The problem of spin-currents with spin-orbit interactions was raised in the context that spin-currents are time-reversal conserving and therefore may possibly be dissipation free [5]. But it was pointed out that not having a conserved current makes such a discussion moot [6]. But it is possible to include the effect of non-conservation of spin and yet define a spin-current operator which does obey a continuity equation [7]. This issue would appear to be important in proper interpretation of spin-Hall effect experiments.

The idea [8] of a state resembling the $\ell = 1$ spin-antisymmetric Pomeranchuk state to Uranium compounds, is not disallowed by the arguments of the papers highlighted above because of the substantial spin-orbit coupling. For URu_2Si_2, where such a state was proposed, experiments have however ruled out such a state; the hidden order is almost certainly the spatial order of a high order multipole [9]. It might however be worthwhile to have Landau type theory generalizing Pomerachuk instabilities to problems with spin-orbit scattering included.

References:
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