Disordered systems, Metastability and the Functional Renormalisation Group

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Consider an elastic line or an elastic membrane embedded in a material containing pinning sites. This model applies to a score of different physical situations, from vortex lines or domain walls to fracture fronts and charge density waves, and leads to a rich phenomenology because of *frustration*: elasticity wants to keep the object straight or flat, whereas the disordered environment tends to distort it in such a way to occupy as many favorable pinning sites as possible. It is intuitively clear that these two competing ingredients are enough to generate metastability: many different microscopic configurations are locally stable, giving birth to a complex energy landscape as often invoked to describe other ill-condensed systems (glasses, spin glasses, proteins, etc.). This metastability leads in turn to a host of fascinating effects specific to disordered systems: non-trivial self-similar roughness, slow dynamics (aging, creep), intermittent depinning (avalanches). A major theoretical difficulty is to devise a formalism able to take into account and describe the diversity of these (sample dependent) metastable states. One example of such formalism is the "Replica Symmetry Breaking" scheme of Parisi, which encodes in a rather magical way the complexity of the spin glass landscape. Another route, devised by Daniel Fisher in the context of pinned elastic objects, is the Functional Renormalisation Group (FRG). The validity, scope, and interpretation of FRG has recently been clarified, after several years of work, by a remarkable series of papers by Pierre Le Doussal, Kay Wiese and associates [1,2,3].

Call $\vec{x}$ the internal coordinates of the elastic object (say, along a vortex line) and $\phi(\vec{x})$ the distortion field. Self-similar roughness means that statistically, $\phi \sim x^\zeta$, where ζ is the roughness exponent. FRG was initially proposed as a way to get around the perturbative "dimensional reduction" result $\zeta = (4 - D)/2$ (where D is the internal dimension, $D = 1$ for a line). Although correct to all orders in perturbation theory, it turns out that this result totally neglects the possibility of different metastable states. The hint of something going wrong appears when one scrutinizes the way the full correlation function of the pinning force, $\Delta_\ell(\phi - \phi')$, renormalizes with scale ℓ – and not only the disorder strength $\Delta_\ell(0)$, as standard RG does. One finds at a finite scale ℓ^*, the second derivative of $\Delta_\ell(\phi \to \phi')$ diverges to infinity;[1] the fixed point of the RG flows must therefore be searched in the space of functions Δ that have a *cusp* at the origin (i.e. an infinite second

[1] The scale ℓ^* is in fact the scale at which metastability appears for the first time. For scales smaller that ℓ^*, called the Larkin length in this context, elasticity dominates and prevents the object from deforming, so that a single ground state is found.

derivative), and not in the space of regular functions. This program, performed by D. Fisher to first order in $\varepsilon = 4 - D$, indeed leads to an exponent $\zeta \approx 0.2083..\varepsilon \neq \varepsilon/2$, with a linear cusp: $\Delta_\infty(u) \approx_{u\to 0} \Delta_\infty(0) - a|u|$. The work of Le Doussal & Wiese extends the above formalism to higher order in ε, an endeavor laden with subtleties which were at first thought to be unsurmountable. Perhaps even more interestingly, their work allows one to set up an intimate test (experimental or numerical) of the FRG formalism by measuring, not only the exponent ζ, but *the whole fixed point function* $\Delta_\infty(u)$, in particular the famous cusp.[2] The idea here is that the correlator of the random force imposed by the disorder on the elastic object can be numerically reconstructed by adding an extra parabolic pinning potential of the form $\frac{m^2}{2}(\phi - \phi_0)^2$ centred around a variable level ϕ_0. For each ϕ_0, one determines the ground state configuration $\{\phi(x)\}_{\phi_0}$ and its center of mass $\phi_{cm}(\phi_0)$. The remarkable result of [1] is that $\Delta_\infty(u)$ is proportional to the correlator of $\phi_{cm} - \phi_0$ for two ϕ_0s chosen a distance u apart, in the limit $m \to 0$. This prescription was used in [2] to compute numerically $\Delta_\infty(u)$ for elastic objects of dimensions $D = 1, 2, 3$ and compare the results to the FRG prediction. The results are spectacular: not only the existence of a linear cusp is confirmed, but the whole function $\Delta_\infty(u)$ is 1% away from the one loop estimate (with a discrepancy well predicted by the two-loop result!).

What is the physics contained in the cusp, and why is it related to metastability? Based on a schematic, $D = 0$ version of the model, Leon Balents, Marc Mézard and myself conjectured in [5] that the cusp in $\Delta_\infty(u)$ is related to *discontinuities* in the effective force as a function of the center of mass position ϕ_{cm}. The physical idea is that as ϕ_{cm} increases, the system switches discontinuously from one favorable metastable state to another at a position ϕ^* where both metastable states have exactly the same energy. Just below ϕ^* the elastic object is dragged backwards whereas just above ϕ^* it is pushed forward, resulting in jump in the force and a stick-slip, avalanche-like motion. The singular $a|u|$ contribution to the correlator $\Delta_\infty(u)$ basically reflects the statistics (density, amplitude) of these discontinuities [5]. In fact, for $D = 0$, the FRG equation maps into the Burgers equation where the velocity is the analogue of the pinning force, and time plays the role of the scale ℓ. The above force discontinuities are the familiar shocks that are formed as the velocity field evolves from a smooth initial condition (i.e. a smooth pinning disorder at the microscopic scale). The work of Le Doussal et al. confirms and makes more precise this analogy: numerically, the 'shocks' in the effective force, responsible for the cusp, appear very clearly [2,3]; analytically, the FRG for $D > 0$ can indeed be written in the form of a 'functional' Burgers equation, meaning that the renormalisation flow does not need to be formulated in terms of correlators of the disorder but *directly in terms of the random pinning force field itself, without any averaging.*

[2] Among the many problems solved by these authors, let us quote the physics of the depinning critical point in the presence of an external force and the description of "droplet" excitations within FRG [4].

These theoretical breakthroughs are important, not only for the problem of pinned systems and their physical properties, but more generally for our understanding of disordered systems. The relation between metastability, shocks in the FRG and Replica Symmetry Breaking can be understood in full details in the context of pinned elastic objects [5,6]. One would hope to find a formulation of the spin-glass problem such that the FRG strategy can be implemented or adapted – this might solve in a particularly elegant way the present replicas vs. droplets deadlock. Finally, the extension of these ideas to treat the role of activated events in glassy dynamics and aging, initiated in [4], would certainly be worth pursuing.

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