Small and large scale granular statics

Authors: Chay Goldenberg and Isaac Goldhirsch

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Recommended with a Commentary by R.P. Behringer, Duke University, Durham, NC, USA

Traditional descriptions of granular materials, used in soil mechanics for instance, posit behavior that is based on intuitive ideas and on certain kinds of experiments, testing for instance, how a granular column responds and ultimately fails under load. From a more fundamental viewpoint, one would like approaches that start at the grain scale and work up to predictions regarding the macroscopic behavior of a granular sample. Granular materials are one member of a class of disordered materials that require new statistical approaches. Others in the same class include foams and colloids, among others.

In this context, there has been much interesting recent work; this commentary considers one such study, by Goldenberg and Goldhirsch[1] (GG) (Granular Matter, 6, 87, 2004), which is arguably one of the most successful at capturing the behavior of recent experiments[2]. Here, the focus is on how forces are carried in static systems. In addition, it is important to make comparison to other recent approaches. Various recent microscopically based models predict dramatically different macroscopic behavior, as characterized by the corresponding PDE that is predicted for the stress. These PDE can be parabolic (diffusive) hyperbolic (wave-like) elliptic (elastic), or in fact of variable type.

GG consider an elastic microscopic description of a granular solid. In the simplest version, the microscopic description is given in terms of particles that interact via simple linear springs, via one-sided linear springs, or more realistic (for 3D) Hertzian contact forces for which the restoring force varies as the deformation/overlap to the 3/2 power. The basic configuration considered by GG is a rectangular sample of spherical particles interacting by various types of springs. Using this type of model, GG consider contact force distributions, the role of force chains in the mesoscopic/macroscopic properties, anisotropy, the role of friction, and the force response function. As with many models, the contact forces are distributed more or less exponentially. From this point of view, the contact force distribution is not so useful as a tool for distinguishing among microscale models (see however below). GG clearly distinguish between force chains, filimentary microscale features which are clearly apparent in 2D photoelastic studies, and the spatially averaged stresses. This distinction is important in light of models where force chains are intimately connected to the larger scale force response of granular sample. Thus, it is the contention of GG that the chains do not *a priori* have any effect above the scale over which they spatially average. Since they choose a scale that is $\sim D$, a particle diameter, their predictions for the response to a point force, necessarily an elastic response, indicate that the force chains need not be seen as a fundamental structural feature for the macroscopic force response of a granular system. The authors then argue that an elastic response, including friction, reproduces recent experimental measurements of the response functions. In very recent work[3] by GG (to appear soon in Nature) these authors further pursue the applicability of elasticity in the force response, and more explicitly the roles of friction and the strength of the applied force.

The story is necessarily more complicated and more interesting. There are at least two recent results of particular interest. Blumenfeld[4] and co-workers, and earlier Witten and Tkachenko[5], have focused on the force response of isostatic granular networks, i.e. systems for which there are just enough contacts to give unique force and torque balance conditions. By contrast, many granular systems have redundant contacts. Isostatic systems are predicted to have a hyperbolic response, and very recently Blumenfeld has shown that the characteristics of the correponding wave equation are the force chains. Briefly, the elastic and hyperbolic pictures can be reconciled by noting that an anisotropic elastic system (which is likely to occur as the isostatic limit is approached) becomes hyperbolic in the extreme anisotropic limit. The interesting question is then whether this transition can be captured either in models or in experiments. The second issue concerns the nominal universal character of the contact force distributions. In recent studies, Snoeijer et al.[6] and Tighe and Socolar (private communicatin) have considered models for how forces are carried in systems with varying degress of anisotropy. Both models predict substantive changes in these distributions as the amount of shear-induced anisotropy increases. This is interesting in at least two regards: first, because increases in anisotropy tend to push the system closer to isostaticity, and second, because such increases in anisotropy are also associated with unjamming, i.e. failure under applied shear.

References:

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