Nonequilibrium Transport in Quantum Impurity Models: The Bethe Ansatz for Open Systems

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Electron transport in nanostructure devices such as quantum dots or quantum wires coupled to reservoirs offers the possibility to study interacting fermion systems far from equilibrium. In a typical stationary nonequilibrium transport situation the reservoirs are in thermal equilibrium, while an electrical current running through the nanostructure drives the local electrons there out of equilibrium.

Well-known examples are the effect of finite bias voltage on transport through quantum dots exhibiting the Kondo effect: with increasing voltage the Kondo enhancement of the conductance is found to be suppressed. This is understood as being a consequence of the gradual destruction of phase coherence as more electrons with high excitation energy (as provided by the chemical potential difference in the two leads) are relaxed through various inelastic processes.

Whereas the conductance of a Kondo dot in the linear response regime may be quantitatively calculated (at least for the simplest models) with the help of the Numerical Renormalization Group (NRG), or other methods such as resummed perturbation theory, local Fermi liquid theory (but not the Bethe ansatz, which has so far not been developed to calculate spectral functions), in the nonlinear regime all these methods fail, with the exception of renormalized perturbation theory. The latter is available for calculating any physical property quantitatively in the regime of "high energies", i.e. provided any of the external field energy scales such as bias voltage V, Zeeman splitting B, temperature T is sufficiently larger than the Kondo temperature T_K. The crossover regime at energy scales $V \sim T_K$ remained out of reach of existing theories so far.

It is therefore of high interest that the paper by Mehta and Andrei presents a new implementation of the Bethe ansatz method, allowing to calculate transport properties. They consider a somewhat simpler model, the interacting resonance level model, connected to two leads. In this model spinless fermions are transported from one lead to the other via a local level, and a fermion on this level interacts with fermions in the leads via a delta function potential.

A successful Bethe ansatz solution has to satisfy two requirements: (1) it should be possible to construct the many-fermion wave functions with the help of the two-particle scattering phase shifts only (i.e. the many-particle phase shifts are zero); (2) the structure of the many-body wave functions is simple enough to allow for the calculation of expectation values of the physical observables. In the traditional Bethe ansatz method the latter condition is essentially only satisfied for thermodynamic quantities, involving the Bethe ansatz energy eigenvalues but not matrix elements, e.g. of the current operator. Therefore the linear response conductance is outside the reach of the traditional Bethe ansatz method.

Mehta and Andrei deal with the above two requirements by constructing the Bethe ansatz many-particle scattering states involving only one-particle and two-particle scattering phase shifts. In fact the two-particle phase factors appear simply as a product over all pairs of particles. These phase factors multiply a product state of single-particle scattering states with N_1 (N_2) particles coming from lead 1 (2). At zero temperature these product states form a filled Fermi sea in lead 1 and lead 2, with separate Fermi energies (chemical potentials differing by the applied bias voltage eV). These states are solutions of the Schrödinger equation. They are not conventional many-body scattering states, however, since the wave function in the incoming channel is distorted by the interaction at the impurity (it involves two-particle scattering phase factors).

On the other hand, this unusual choice of scattering states has big advantages: it allows for a simple calculation of expectation values of physical operators, in this case the charge current and the occupation number of the local level. These quantities turn out to be given by their (single-particle) expressions in the absence of interactions, with momentum eigenvalues renormalized by the interaction.

It remains to determine these renormalized Bethe ansatz momenta. This can be done by the usual momentum quantization procedure: a given momentum eigenvalue is shifted by the effect of two-particle scattering phase shifts due to all other occupied states. The resulting integral equations for the two densities of occupied states in lead 1 and 2 are linear and depend on the occupation numbers in the leads. Actually the equations given in the paper are not quite complete, as they do not contain the contributions from bound states. As a consequence, the numerical evaluations presented in the figures in the paper cited are probably not correct.

The method presented in this paper for calculating transport properties of quantum impurity models in stationary nonequilibrium represents a considerable advance and will lead to a new generation of exact solutions of transport problems in nonequilibrium—provided it is correct. There is one highly unusual feature of this model, which is essential for its "integrability": the single-particle scattering wave functions have to be chosen in a particular way, which makes them discontinuous at the impurity site. The value of the wave function at the impurity site is adjusted such that certain unwanted terms in the two-particle and the many-particle wave function drop out. Mehta and Andrei justify this choice by arguing that the continuum model they are using is not uniquely defined. What they have in mind, we believe, is that the delta function interaction at the impurity needs to be regularized. Provided a particular regularization leads to the desired discontinuous wave function, the model would be well defined and the conclusions drawn by Mehta and Andrei would be unescapable. At present it is not known whether such a regularized version of their model can be constructed.

For the moment the proposal of a Bethe ansatz for open systems remains an exciting possibility.