Universal scaling relation in high-temperature superconductors.

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Recommended and a Commentary by P.W. Anderson, Princeton University.

The Uemura relation, $T_c \propto \rho_s$, although fitting quite well in the underdoped region of high T_c materials, fails badly for optimal and over doped materials. Though it is often discussed in the context of "ordinary" superconductors, it can hardly hold there since ρ_s is impurity sensitive and T_c is not; still less can it possibly hold for c-axis superfluidity, since ρ_s (c) varies wildly. Where Uemura fits it seems to be compatible with the Lee-Wen idea that superconductivity is destroyed by thermal excitation of quasiparticles.

Homes et al seem to have found a much more nearly universal relation, fitting as well as Uemura where the latter works, and covering also the higher dopings, and, rather amazingly, the c-axis electrodynamics as well as that on the ab plane. They achieve this miracle by dividing the superfluid density by the DC conductivity at the transition temperature T_c, usually obtained from extrapolation of the infrared conductivity. (This step isn't questionable; direct measurements agree when they exist.) A miracle, to my mind, is that this DC conductivity seems to be constant with doping x in the underdoped regime, even though ρ_s is fairly accurately proportional to x, and even though the resistivity has no simple variation with T.

The quotient {(superfluid density)/conductivity} defines a relaxation rate $1/\tau$, obtained by assuming that the superfluid density contains all the mobile electrons, and that they all relax at the same rate $1/\tau$. More formally, this is the result obtained if the conductivity is described by the simplest bubble diagram without vertex corrections, using renormalized single-particle propagators $G(k,\omega)$ and assuming that the IP of the single-particle self-energy is h/τ in the normal state, but the self-energy is purely real in the superconductor. This also amounts to saying that all of the resistivity comes from quasiparticle decay due to strong electron-electron interaction, the familiar basic assumption of either the marginal Fermi-Liquid or the non-FL theories of transport in the normal state--see my book. If this is the case it is interesting, if not original, that the c-axis τ, defined this way, should be the same as that defined from the ab plane--as pointed out long ago by N Kumar and collaborator.

What is new, and very germane, is that the scaling law then claims that $k_B\, T_c$ = (const) x h/τ. If my estimate is right the numerical factor is 1.4 if one uses h-stroke. This is non-trivial, and must tell us something fairly fundamental about the mechanism for T_c.

What must be a coincidence gives Homes et al an equivalent, and actually well-known, relationship in the extreme dirty limit of BCS superconductors, though they note that the coefficient is about a factor of 2 different (It's to be found in Tinkham's book, I believe.) But it fails to hold for the clean limit by many orders of magnitude, which must be true since T_c and ρ_s are dirt-independent and resistivity not--and of course the mechanism depends not at all on dirt.

There are two accepted kinds of superconducting transition, which until now have been assumed to be the extremes of a one-dimensional manifold where the parameter is the ratio of pair binding energy (the "gap", more or less) to the single particle kinetic energy E_f. Neither of these obeys anything like Homes' relation (taking the clean limit as more meaningful for the BCS case). For a Bose liquid of bound pairs, T_c is the degeneracy temperature, proportional to n in one dimension, and h/τ would be n to the 3/2 power for particle collisions, n to the 1/2 for dirt.

I conjecture that the high T_c transition is of a qualitatively different type and does not belong in the one-dimensional manifold described above. This T_c occurs when the quasiparticles break apart: it is the deconfinement transition, possibly deconfinement of separate charge and spin excitations; but in any case condensation takes place when the electron as a quasiparticle first begins to have an energy definition comparable to its mean energy $\Delta E=k_B T$.