Frustrated Polyelectrolyte Bundles and T = 0 Josephson-Junction Arrays
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Despite our intense fascination with electromagnetism, from the Spring of our first year of college through *Classical Electrodynamics*, there are still many surprising phenomena and effects, especially when combined with statistical mechanics. In salt solution, the standard theory of electrostatics starts with the self-consistent nonlinear Poisson-Boltzmann equation which, when linearized, gives the standard Debye-Hückel theory with exponentially-screened interactions between charges. However, there are significant deviations from this general screened repulsion, particularly when the counterions are multivalent and strong correlations between the screening charges become important. Indeed, even when considering parallel, uniformly charged rods, Shklovskii [1] argued that the counterions form a Wigner crystal along each rod at zero temperature. Thus, if the parallel rods are on a two-dimensional, bipartite lattice (*i.e.* one that can be split into two regular sublattices), it is then possible to arrange a phase shift of the one-dimensional Wigner crystals between the two sublattices (along the rod direction) to form an attractive, three-dimensional lattice of counterions among the forest of parallel rods.

This is not just a theoretical curiosity, however. DNA, a potent polyelectrolyte, forms densely-packed bundles in the presence of multivalent salts. This then begs the question of how the charges arrange themselves when the charged rods are in a triangular lattice. Though it may be no surprise that some frustrated phase shifting occurs between the individual Wigner crystals, Grason and Bruinsma have elegantly shown that this problem can be mapped directly onto a zero temperature, frustrated ($f = 1/2$) Josephson junction array, studied by Kosterlitz and Granato [2]. In the polyelectrolyte problem, the two competing interactions are the strength of the interaction which favors a π phase shift between lattice sites and the stiffness of the Wigner crystal on each rod. In their mapping, these become the inverse of the capacitive charging energy of each grain and the inter-grain coupling, respectively. More importantly, the stiffness of the resulting three-dimensional counterion lattice is precisely the current-current correlation in the corresponding Josephson junction array (for imaginary frequency, but only egoists count i's). Much is known about the quantum system and the zero-temperature phase transitions as the ratio of the lattice stiffness to the interaction strength is varied. When either the crystal stiffness or the interaction is weak, the system is in the insulating phase and there is no long-range, three dimensional order. However, when either of these couplings grow (technically, *their product*), the counterion lattice orders as in the superconducting state of the junction array. This changes the elasticity from that of a columnar phase, a two-dimensionally ordered liquid crystalline state with a bending modulus but with some vanishing shear stiffness, to that of a true three-dimensional crystal. The ordering of the counterions is helical and the added Z_2 degeneracy of the XY antiferromagnet allows for either right- or left-handed "helical" stacking as in HCP lattices (*i.e.* $ABCABC\ldots$ or $ACBACB\ldots$).

Interestingly, at the critical point, the current-current correlation function is nonanalytic in the momentum, *i.e.* $\propto |p|$ [3], suggesting a novel, intermediate phase between the

crystal and the columnar phase with vanishing shear moduli but infinite bending energy. This intervening phase is, apparently, what is necessary to render the crystal to liquid crystal phase transition second instead of first order. The authors hypothesize further connections between the topological melting in the junction array and the corresponding topological mode of melting in the counterion lattice. This work is surely the seed of important and prolific future work.

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[1] B.I. Shklovskii, Phys. Rev. Lett. **82** (1999) 3268.
[2] E. Granato and J. M. Kosterlitz, Phys. Rev. Lett. **65** (1990) 1267.
[3] M. P. A. Fisher, G. Grinstein and S. M. Girvin, Phys. Rev. Lett. **64** (1990) 587.