Efimov Effect and Renormalization Group Limit Cycle

Evidence for Efimov quantum states in an ultracold gas of cesium atoms

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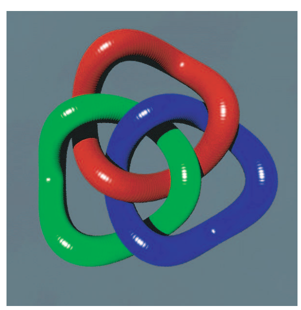
Recommended and Commentary by Tin-Lin Ho, Ohio State University

In 1970, Efimov (*Phys. Lett. B***33**, 563 (1970)) pointed out the remarkable fact that even when a two-body potential is so weak that it can not accommodate a two-body bound state, *if it is close to resonance*, (i.e. a state in the continuum is about to come down to form a bound state), such potential can generate *infinitely* many three-body bound states with energy $-|E_n|$ accumulating toward the continuum, (i.e. $E = 0$), with ratio $|E_{n+1}/E_n| = e^{2\pi s_o}$, $s_o = 1.00624$. This effect is often liken to the Borromean rings (see figure), which is a set of three interlocking rings such that if one is removed, then the other two will fall apart. The formation of this stable structure requires the cooperation of all three rings.



Although the Efimov effect has been a fascination among nuclear and atomic physicists for decades, experimental confirmation has been lacking because it is hard to find particles whose interactions are close to resonance. The situation, however, changes dramatically in recent years. Due to the recent realization that atomic interactions can be controlled using Feshbach resonance, one now can tune the two-body potential to resonance by simply changing a background magnetic field. In addition, Hammer and Braaten have pointed out recently (*Phys. Rev. A***70**, 042706 (2004)) that the energy spectrum of the Efimov states can lead to a periodic variation in three-body decay rate. Not only can this variation be used to identify Efimov states, but also to locate magnetic fields at which three-body decay rates are small, which is very important for producing stable quantum gases. Efimov effect was finally found by Rudi Grimms group at Innsbruck (T. Kraemer et.al. *Nature* **440**, 315 (2006)), and that the prediction of Hammer and Braaten was well verified.

I shall not discuss the discovery of Efimov states because it was covered very nicely by Charles Day in *Search and Discovery* of *Physics Today*, April 2006. Instead, I would like to point out an interesting but less emphasized connection between Efimov effect and condensed matter physics, i.e. that it is an example of the Renormalization Group limit cycle. In his famous 1971 paper (*Phys. Rev. D***3**, 1818 (1971)) on renormalization group (RG), Ken Wilson pointed out that the RG equations can have solutions other than fixed points, such as limit cycles. In condensed matter physics, however, there have been very few examples of RG cycles. It is therefore interesting to find out more examples of this phenomenon. That the Efimov effect is a RG limit cycle was realized quite sometime ago by S. Albeverio et.al. (*Phys. Lett.* **83A**, 105, (1981)), and has recently been studied extensively by S.D. Glazek and K.G.Wilson (*Phys. Rev. Lett.* **89**, 230401 (2002), ibid **92**, 139901 (2004), *Phys. Rev. B***69**, 094304 (2004)).

The basic physics of the Efimov effect is that when the two-body potential is at resonance, if one averages out one of particles in this three-body system, the effective interaction between the two remaining particles is an attractive $1/r^2$ potential, with a cutoff r_o that depends on short range details of the two-body potential. The Schrodinger equation of the remaining two particles (with distance r) is

$$\frac{\hbar^2}{2m}\left[-\frac{\partial^2}{\partial r^2}-\frac{s_o^2+1/4}{r^2}\right]f(r)=Ef(r), \tag{1}$$

with the boundary condition $[(r\partial_r f)/f]_{r_o}=G(r_o)$, where $G(r_o)$ is the logarithmic derivative specifying the behavior at short distance.

Since $1/r^2$ scales as the kinetic energy, the Schrodinger equation is scale invariant. It is this invariance that leads to infinitely many bound states accumulating to continuum. Keeping the spectrum fixed, one can work out how changes with respect to rescaling of . One then finds that varies periodically as , which is a limit cycle solution of the RG equation. If the exponent 2 in $1/r^2$ is changed to a different power, the limit cycle will turn into a fixed point.

Since many phenomena in condensed matter are described by RG equations with fixed point solutions, it is natural to speculate that there might be many more systems whose physics are described by RG limit cycles. Although we only have a few examples at present, phenomenon such as the Efimov effect is sufficiently intriguing that it only raises the curiosity.