Berezinskii-Kosterlitz-Thouless Crossover in a Trapped Atomic Gas

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Recommended and Commentary by Steven M. Girvin, Yale University

The Berezinskii-Kosterlitz-Thouless phase transition is a novel (and quite fundamental) phenomenon in which a classical statistical mechanical system undergoes a continuous ordering phase transition without the appearance of a local order parameter, that is, without spontaneous symmetry breaking. It occurs in two spatial dimensions in a broad class of systems whose fluctuations are described by a complex (i.e. two-component) field with an internal XY rotational symmetry. Important realizations include the superfluid transition of helium films adsorbed on surfaces, superconductivity in amorphous thin films, the melting of two-dimensional solids, and the ordering of two-dimensional magnets with easy-plane anisotropy.

The work of Hadzibabic et al. now adds to this list an important new system, ultra-cold atomic gases confined in traps and manipulated by optical lattice potentials. This new work demonstrates a remarkable means to directly visualize the quantized vortices whose unbinding characterizes the phase transition.

Mean field theory based on a phenomenological Ginsburg-Landau free energy functional is often a good starting point for understanding ordering phase transitions. However in lower spatial dimensions, fluctuation effects beyond mean field theory become increasingly important. The Hohenberg Mermin Wagner theorem says that in dimensions $d \leq 2$, fluctuations are so large that any ordering which breaks a continuous symmetry is destroyed by thermal fluctuations. This can be seen within a simple spin wave theory based on gaussian fluctuations of the order parameter field, which shows that the field correlation function falls to zero algebraically with distance. Thus the net magnetization of an XY magnet is zero at any finite temperature. On the other hand, general arguments (backed up by high temperature series expansions) require that the correlation function fall off exponentially at high temperatures. Hence, there must be a phase transition at some temperature T_{BKT} below which the decay is algebraic and above which it is exponential. The magnetization is zero in both phases and yet there is a phase transition!

The key to understanding this is the realization that in addition to smooth spin-wave fluctuations around the perfectly ordered state, there are also vortex topological defects in which the order parameter phase winds by $\pm 2\pi$ along paths encircling the defect. Examination of the energy stored in such a defect shows that it diverges logarithmically with system size. Hence at low temperature, vortices of opposite winding number are confined together by an attractive logarithmic potential. At high temperatures, the entropy (which is also logarithmic in system size) wins and the free energy of unconfined vortices becomes favorable. The proliferation of these topological defects produces the exponential decay of the correlation functions.

Hadzibabic et al. directly observe these topological defects via the quantum interference of matter waves emanating from a bilayer pair of two-dimensional cold atomic gases. At zero temperature, releasing the two gases from their confining potentials leads to a two-slit interference pattern in the matter waves. Because the two gases have been separated by a large barrier, they have not been in communication and their overall relative phase is random. This does not destroy the two-slit pattern; it simply gives it a random offset in position. At finite temperatures, the presence of a vortex in one layer or the other produces a 2π phase slip that makes its appearance as a 'screw dislocation' in the matter wave density pattern as schematically illustrated in the figure below.

Amusingly, this result has deep connections with two seemingly unrelated problems. First, optical vortices (phase windings of the electromagnetic field around the line of propagation) are routinely generated holographically by diffracting light from gratings containing various types of singularities such as fork dislocations. Here we are dealing with a three-dimensional helical grating containing a screw dislocation. If one were to Bragg scatter light from this matter wave interference pattern, one would map the BKT vortices onto vortices in the optical field.

Second, in the melting of two-dimensional crystals, it is the appearance of fork dislocation defects in the crystal which produce exponential decay of positional correlations of the atoms. Ironically, the usual theoretical modeling of these dislocations consists of inventing a mapping onto an order parameter field that represents them as logarithmically interacting vortices. Hence their thermal proliferation is in the BKT universality class. Thus, in modern light, we now understand that the complex order parameter field used to describe these dislocations is essentially just the optical field that would result from diffracting light from the solid.

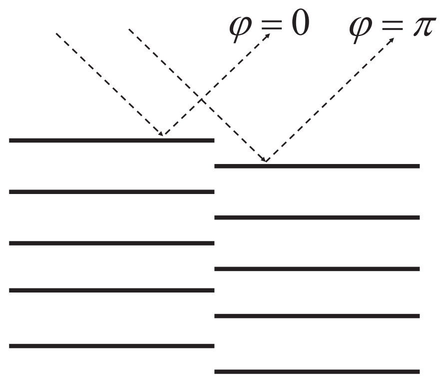

Figure 1: Edge view showing the 2D projected density of the matter wave interference pattern resulting from the presence of a vortex in one of the two original condensate layers. On the left the relative phase of the two condensate layers is zero. On the right it is π, resulting in a shift of the interference pattern by half a lattice constant. The full three dimensional density profile of the interference pattern is a helix analogous to that of a screw dislocation in a crystal. Dashed lines indicate an optical wave being Bragg reflected from the matter wave. The phase variation of the optical field across the sample would result in the creation of an optical vortex in the scattered light.

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