Generalized Bose-Einstein phase transition in large-m component spin glasses.

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Recommended with a Commentary by Subir Sachdev, Yale University

A description of the low temperature phase of finite dimensional spin glasses has remained one of the central open problems in statistical mechanics for several decades. There are several competing models, the most prominent among them being the theory of replica symmetry breaking, and the droplet model. The latter approach has a direct physical interpretation, but has so far been phenomenological in approach, and has not offered a framework for quantitative microscopic computation.

Aspelmeier and Moore propose to address this last shortcoming by re-examining the large-m limit of an m-component spin glass, and find some surprising complexity that has so far been overlooked.

The physical idea goes back to early work of Hertz, Fleishman, Anderson (Phys. Rev. Lett. 43, 942 (1979)) and Bray, Moore (J. Phys. C 15 L765 (1982)). Consider the onset of spin-glass order from the high temperature paramagnetic state. One can view the initial order as a condensation into an eigenmode of a certain susceptibility matrix with the smallest eigenvalue. However, these low-lying eigenstates are inevitably localized, and the large repulsive interaction between `bosons' occupying the same mode pushes the eigenvalue of the localized modes higher as one approaches the transition. At the critical temperature, the localization length of the lowest eigenmode diverges, and a condensation transition becomes possible. This is an elegant example of the interplay between localization and interactions, and has offered a valuable perspective on other systems over the years: similar ideas have been used near quantum critical points (Hartman and Weichman, Phys. Rev. Lett.74, 4584 (1995)) and in recent work on the onset of magnetic order in superconducting vortex lattices.

Aspelmeier and Moore extend this approach to very low temperatures within the spin glass state, a regime which, surprisingly, has not been studied so far. They consider a finite size system with N spins, and formally look at systems with m >> N. Their main finding is that the lowest eigenmode acquires a large degeneracy in the limit of large N, with the degeneracy scaling roughly as the square root of N. These degenerate modes presumably capture the complexity of the spin glass state. The main advantage of the present approach is that this complexity can be studied directly in finite dimensions, and for any specific microscopic model.

This work suggests many additional questions that need to be addressed. What is the significance of the peculiar manner in which the thermodynamic limit was taken? What is the role of the large number of Goldstone modes present for large m? What is the nature of the 1/m corrections, and do they lead to an improved understanding of the critical point?

NOTE ADDED 11/25/03: The existence of a large number of low-lying eigenmodes in the infinite range, large-m spin glass was studied earlier by M. Hastings, J. Stat. Phys., 99, 171 (2000).