Spin glasses: the saga continues!

Recommended with a Commentary by JP Bouchaud, CEA, Saclay

The debate over the nature of the spin-glass phase in finite dimensions (and in experimental samples, for that matter) is still vibrant, as illustrated by a series of preprints posted on cond-mat since the beginning of the summer [1,2,3,4,5].

Ever since the controversy started in the mid-eighties, the basic question concerns the nature of the symmetry that is broken by the spin-glass transition. Since the spin-glass phase is thought to be characterized by the appearance of a non zero (but random) local magnetisation, the simplest idea is that the broken symmetry is the standard up-down, Ising symmetry. The spin-glass would thus merely be a disguised ferromagnet -- as surmised in the droplet theory -- although a rather well disguised one. The presence of rare, low energy excitations of arbitrarily large size, confer critical properties to the spin-glass; in particular the non-linear susceptibility diverges in the whole spin-glass phase. However, because the broken symmetry is the up-down symmetry which is sensitive to the magnetic field, there cannot be a strict phase transition in non-zero field, only a dynamical crossover.

In mean-field, on the other hand, things turn out to be much more complicated. The broken symmetry is not the Ising symmetry but rather Parisi's abstract `replica symmetry', which encodes mathematically the presence of many possible ordered phases, unrelated to one another by simple transformations. Whereas a magnetic field is enough to select one phase out of two, it does not help much if there is a very large number of possible frozen configurations towards which the system can evolve. Therefore in mean-field one expects, and indeed finds, a true phase transition even in non zero field -- the celebrated de Almeida-Thouless (AT) line.

What survives of Parisi's replica symmetry breaking scheme in finite dimensions? Is there a critical dimension above which mean-field is essentially correct (and where the AT survives as a true phase transition), but below which replica symmetry is restored, and the droplet picture is suitable? This question should in principle be answered using a replica field theory, where spatial and replica degrees of freedom are treated in a consistent way. However, the theory is fraught with technical difficulties, and real progress has been slow. For example, the stability of the replica symmetric solution in finite dimension d can be investigated by computing the mass of the propagators of the theory. In high dimensions, there seems to be no doubt that the replica symmetric solution is unstable: the mass of propagators can be computed and are found to be both finite and negative. Surprisingly, in $d < 6$, the situation seems to be even worse for the replica symmetric solution because the mass diverges negatively as the zero-replica limit $n \to 0$ (needed to reach the spin-glass physics) is approached. This is strange because one would expect the droplet theory to be relevant for low dimensional spin-glasses (spin-glass chains are not even frustrated!) and that the instability would decrease, not increase, as d decreases. Conversely, Parisi's replica symmetry broken solution appears to be stable in $d > 6$ but some divergencies in fact appear for $d < 6$.

A way out of this quandary was proposed by M. Moore in [1]. He proposes that the apparent divergence of the mass for $d < 6$ when $n \to 0$ is only superficial, in the sense that if one could resum the perturbation theory in the amplitude of the non linear terms in the replica field theory, one would finally end up with a *finite* result when $n \to 0$. Whether or not the mass is negative would now depend on the value of this finite contribution, but the replica symmetric solution could possibly save its head in $d < 6$. Although Moore cannot show that this scenario really holds in the case at stake, he analyzes an exactly solvable toy problem where perturbation theory has the same structure. It is divergent term by term when $n \to 0$, but an exact resummation can be performed and eventually leads to perfectly sensible results. There is obviously a large gap between this toy problem and the formidable complexity of the spin-glass field theory, and Moore's conclusions have been recently challenged by Temesvari [4]. However, several independent results suggest the absence of an AT line below $d < 6$ [1], such as the beautiful numerical work of Katzgraber and Young on spin-glass chains with long range interactions [2]. Depending on the shape of the long-range interactions, the spin-glass chain can be either in a mean-field like regime, or in a regime similar to what is expected for true spin-glasses below the critical dimension. The main finding of this paper is that an AT line is clearly detected in the former, mean-field regime, but seems to disappear in the latter regime corresponding to $d < 6$. A careful analysis of experimental data also suggests the absence of an AT line for Ising spin-glasses in d=3 [6]. The divergencies associated to Parisi's replica symmetry broken solution in $d < 6$ would be another indication of the change of nature of the spin glass phase in low dimensions [1]. The (temporary?) consensus seems to be that for $5/2 < d < 6$, spin-glasses have a finite transition temperature T_c but are replica symmetric -- the value 5/2 for the lower critical dimension is supported by several numerical results compiled and analyzed by Boettcher [3] (but to add to the confusion it is precisely the value predicted by a replica symmetry breaking calculation!).

Many physical questions remain, beyond the technical ones raised in [4]. For example, how can all the subtleties of the droplet picture (rare, large zero energy clusters, temperature chaos, etc.) be consistently encoded in a supposedly `simple' replica symmetric field theory? [5]. Conversely, how can one construct a phenomenological picture of low lying excitations in spin-glasses compatible with the existence of an AT line? Certainly, the non-compact nature of excitations in high dimensions should play a role. It is precisely because tenous extended structures can pass through each other without interacting much that Parisi's many state may make sense in high dimensions [7].

[1] M. A. Moore, The stability of the replica symmetric state in finite dimensional spin glasses, http://ArXiv.org/cond-mat/0508087

[2] H. Katzgraber, P. Young, Probing the Almeida-Thouless line away from the mean-field model, http://ArXiv.org/cond-mat/0507138

[3] S. Boettcher, Stiffness of the Edwards-Anderson model in all dimensions, http://ArXiv.org/cond-mat/0508061

[4] T. Temesvari, Is the droplet theory for Ising spin glasses inconsistent with replica field theory? http://ArXiv.org/cond-mat/0510209

[5] The recent paper of C. De Dominicis, W-T identities and a candidate "droplet" Lagrangean for the Ising Spin Glass, http://ArXiv.org/cond-mat/0509096, is perhaps an attempt in that direction.

[6] P. E. Jonsson, H. Takayama, H. Aruga Katori, A. Ito, Dynamical breakdown of the Ising spin-glass order under a magnetic field, cond-mat/0411291, Phys. Rev. B 71, 180412(R) (2005)

[7] see: O. White, D. S. Fisher, Scenario for Spin Glass Phase with Infinitely Many States, http://ArXiv.org/cond-mat/0412335