Scale invariance and universality of force networks in static granular matter

Authors: Srdjan Ostojic, Ellak Somfai and Bernard Nienhuis

Nature, **439**, 828 (2006)

Recommended with Commentary by R.P. Behringer, Duke University

How forces are carried in dense granular media is a topic of considerable interest and importance. Much attention has been given to the distribution of contact forces between pairs of particles[1]. But much less attention has been given to spatial structures. Photoelastic images in 2D in particular[2], as well as various forms of molecular dynamics simulations[3] show very clear filamentary structures, so called force chains. These networks carry the largest forces throughout the packing; and their exist networks of particles that bear lower forces. An important issue is then how to characterize the spatial structures that seem so evident to the eye. There has been some work that addresses the details of the force distributions in space[2,4], but much remains to be learned. One approach is to focus on the fact that force chains have a linear aspect, and to obtain the statistical properties of chain segments, as done recently for instance by Peters et al.[4]

In their paper, Ostojic et al. analyze model results for static materials with an eye to characterizing force clusters, rather than force 'lines'. This provides a natural connection to percolation phenomena. Contacting grains define a bond, with force f. Clusters consist of contacting grains with bond forces above some threshold in f. These authors then test a scaling model for the distribution of cluster sizes, s, of the form $P(s, f) = s^{-\tau}\rho(s/(f - f_c)^{\sigma})$. Here, f_c is a critical force threshold. More particularly, they consider scaling of the moments of $P(s, f)$ as a function of f and of the system size, as defined by the number of contacts, N, in the system. Scaling of the nth moment, m_n of $P(s, f)$ is then expected to vary as $m_n = N^{\phi_n} M_n([f - f_c]N^{1/2\nu})$, where the exponents ϕ_n and ν are algebraic functions of n, σ and τ.

Ostojic et al. find that for a collection of systems, there is apparently universal collapse of $N^{-\phi_2} m_2$ onto a scaling form, which defines the scaling function M_n. The systems for which scaling works includes MD models of isotropically compressed frictional particles, and also an Edwards entropybased model originally proposed by Snoeijer et al.[5] Details of the interparticle interactions used in the MD simulations affect f_c but not the scaling. However, different models, including the q-model[1] and MD models with shear-induced anisotropy do not yield the same kind of scaling.

This approach seems particularly appealing, since it does not need to invoke a specific geometric structure, such as force chain segments. The fact that it seems to work for 'classes' of systems is also appealing and may help to identify important properties and members of classes. The authors speculate that a similar approach may work in 3D, but that clearly must be tested. Finally, the fact that this approach does not seem to work for sheared systems implies a need for more understanding that, one hopes, can generalize this approach.

References

[1] S. N. Coppersmith, C.-h. Liu, S. Majumdar, O.Narayan and T. A. Witten, Phys. Rev. E, **53**, 4673 (1996).

[2] T. Majmudar and R. P. Behringer, Nature **435**, 1079 (2005).

[3] F. Radai, J.-J. Moreau and S. Roux, Phys. Rev. Lett. **77**, 274 (1996).

[4] J. F. Peters, M. Muthuswamy, J. Wibowo and A. Tordesillas, Phys. Rev. E **72**, 041307 (2005).

[5] J. Snoeijer, T.J. H. Vlugt, M. van Hecke, and W. van Saarloos, Phys. Rev. Lett. **92**, 054302.