Topological Band Insulators in two and three dimensions in spin-orbit coupled systems.

A. Quantum Spin Hall Effect in Graphene
Authors: A. C. Kane and G. Mele
http://arXiv.org/cond-mat/0411737
Phys. Rev. Lett. **95**, *226801 (2005)*

B. Quantum Spin Hall Effect and Topological Phase Transition in HgTe Quantum Wells
Authors: A. Bernevig, T. Hughes, S.C. Zhang
http://arXiv.org/cond-mat/0611399
Science, 314, 1757 (2006)

C. Topological invariants of time-reversal-invariant band structures
Authors: J. E. Moore, L. Balents
http://arXiv.org/cond-mat/0607314
Phys. Rev. B 75, 121306(R) (2007)

D. Topological Insulators in Three Dimensions
Authors: D. Liang Fu, C.L. Kane, E.J. Mele
http://arXiv.org/cond-mat/0607699
Phys. Rev. Lett. **98**, *106803 (2007)*

Recommended and Commentary by Ashvin Vishwanath, UC Berkeley

It is commonly believed that the band insulators are particularly simple and well understood states. A new variety of band insulator emerged with the discovery of Chern insulators (Integer Quantum Hall state), characterized by their quantized Hall conductivity or, equivalently, the number of chiral (one way propagating) edge modes. Realizing these states however requires the breaking of time reversal symmetry (eg. a magnetic field).

The recent theoretical progress reviewed here predicts the existence of novel types of band insulators *with* time reversal symmetry, when the spin-orbit coupling is present. These insulators appear as stable phases distinct from the conventional insulator in both two and three dimensions, but their classification is more subtle than that of Chern insulators since they are not associated with a quantized response. These non-trivial insulators, or Topological Band Insulators, have edge or surface states that are protected and provide interesting realizations of one and two dimensional free fermions systems respectively that can remain delocalized in the presence of (non-magnetic) disorder. They have a two fold (or Z_2) classification; non trivial insulators can be distinguished from the trivial insulators by the existence of an odd number of protected surface modes, while the case of even number of surface modes is not similarly protected. Since spin-orbit coupling effects are widespread, the prospects for experimental realization are very bright with some serious candidates already identified. Given the interest in spin based electronic devices, and the conceptual novelty of these states, we are sure to see much more experimental activity on them in the future.

2D Topological Band Insulator: In Ref. A, a simple realization of the 2D topological band insulator was given based on the graphene band structure. With just the nearest neighbor hopping Hamiltonian on the honeycomb lattice one obtains Dirac points, but adding to this the most natural spin orbit interaction term (an S_z spin dependent second neighbor hopping), a gap opens leading to an insulator.

$$H = -t \sum_{\langle ij \rangle} \psi^\dagger_{i\sigma} \psi_{j\sigma} + it_2 \sum_{\langle\langle ik \rangle\rangle} v_{ij} \psi^\dagger_{i\sigma} \sigma^z_{\sigma\sigma'} \psi_{k\sigma'}$$

Remarkably, this insulator automatically has the up-spins in the $+1$ and the down spins in the -1 integer quantum Hall state. At the edge, there is a pair of counter-propagating edge modes, with the up spin circulating in one direction, and the down spin in the opposite direction. Since this combination preserves time reversal symmetry there is no Hall effect, however there is a quantized 'spin-hall' effect where an electric field induces a transverse spin current. In general, counter-propagating modes are unstable since they can mix and split, leading to a gap at the edge. It might appear here that it is the S_z spin rotation symmetry that forbids the up and down spin edge states from mixing. An important insight in Ref A was that the edge states are stable even when S_z is not conserved, under the much weaker assumption of time reversal symmetry. To see this, try writing an operator that mixes the Right and Left moving edge states: $\mu \psi^\dagger_{R\uparrow} \psi_{L\downarrow} + \mu^* \psi^\dagger_{L\downarrow} \psi_{R\uparrow}$ but this is odd under time reversal: $\mu \to \mu^*$, $\psi_{R\uparrow} \to \psi_{L\downarrow}$, $\psi_{L\downarrow} \to -\psi_{R\uparrow}$ and hence forbidden. The same peculiar algebraic structure that gives rise to the Kramers degeneracy is operative here as well. Although the spin-hall effect is no longer quantized once spin symmetry is broken, it is clear that one is dealing with a new class of insulator. An even number of counter-propagating edge pairs though is not protected by just time reversal symmetry; hence this is a Z_2 classification of insulators.

Experimental Realization: In Ref A, graphene as suggested as a possible system to realize this state, but has the drawback of weak spin orbit coupling.

A significant step forward towards a more practical realization was suggested in Ref B, of a CdTe/HgTe semiconductor quantum-well structure which was realized very rapidly in experiments [1], which have been reviewed separately in a Dec 07 Journal Club article.

3D Topological Band Insulator: A further theoretical surprise was the discovery of a three dimensional version in Ref C (and [2]), that has no quantum Hall analog. In Ref D the remarkable properties of this 'strong' topological insulator phase are described. The strikingly new feature is again the existence of surface modes protected by time reversal symmetry- in this case the two dimensional surface of the 3D insulator is endowed with an odd number of protected Dirac nodes. A feature of two dimensional systems is that Dirac nodes always occur in pairs (recall graphene). The strong topological insulator beats these Dirac point doubling requirements by creating a two dimensional system on the surface of a three dimensional insulator. While the Dirac point is not generally at the Fermi potential, the presence of an odd number of Dirac nodes still has striking consequences. Model band structures based on the distorted diamond lattice which may be relevant to materials like BiSb alloys were proposed in Ref D to be in this phase. Unlike the 2D case, where it is easy to understand the topological band insulator beginning with a model system with a conserved spin component, spin conservation is necessarily absent in the 3D case. Nevertheless there are points of close similarity. Just as the edge of the 2D insulator may be viewed as containing half the modes of a one dimensional wire, with the other set banished to the opposite edge, similarly, the half of the Dirac points of a conventional 2D band structure live on one surface of the 3D insulator, with the other half on the opposite surface.

Several future directions suggest themselves - one is the interplay between various orders eg. magnetism and superconductivity and the topological band insulator. For example, in [3] it was shown that the superconducting proximity effect on the surface of a 3D topological band insulator leads to vortices with non-abelian statistics, which might eventually be the most practical way of generating these exotic states. New experimental signatures of these states are required, as well as theoretical descriptions analogous to the Chern Simons description of integer quantum Hall states.

[1] M. Konig, et al. Science 318, 766 (2007)
[2] R. Roy arXiv:cond-mat/0607531
[3] L. Fu and C. Kane arXiv:0707.1692