Magnetorotational-type Instability in Couette-Taylor Flow of a Viscoelastic Polymer Liquid
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Recommended with a commentary by Wim van Saarloos, Instituut-Lorentz, Leiden University

Cosmology and astronomy are observational fields — one may maximally explore the implications of observations that are being mined from the enormous variety of telescopes, and try to extrapolate back in time, but in most cases (astrochemistry is one exception) one lacks the possibility of doing a controlled experiment in the lab. For this reason it is always interesting and stimulating if strong analogies are found between cosmological or astronomical phenomena on the one hand and condensed matter systems on the other hand, as the latter can be explored in the lab. For instance, the symmetry breaking phase transitions which are thought to have taken place in the early universe and to have given rise to the formation of topological defects, have many analogies with symmetry breaking phase transitions that take place in liquid Helium and Bose condensates. Between 2001 and 2006, there was even a specific European program *Cosmology in the Laboratory* (COSLAB) to explore these similarities. Apart from this, it is almost always intellectually rewarding too to explore analogies between very disparate fields — a well known example is the similarities between smectic liquid crystals and superconductors that de Gennes used to his advantage.

In the paper *Magnetorotational-type Instability in Couette-Taylor Flow of a Viscoelastic Polymer Liquid* the authors point at another, even simpler, type of analogy that opens the possibility to study the basics of a magnetorotational instability that arises in astrophysical magnetohydrodynamics in the lab, using polymers at room temperature!

The astrophysical instability that the authors address occurs in a differentially rotating fluid of sufficiently high electrical conductivity in the presence of a magnetic field. Apparently, this instability, which occurs when the angular velocity decreases with the distance from the axis of rotation, is especially relevant to astrophysical accretion disks, where it is believed to give rise to turbulence. It may also occur in stellar interiors.

The essence of the instability is the following. Since magnetic field lines do not end (no monopoles) they act somewhat like elastic tethers. Now, consider two fluid elements connected by a magnetic field line, both at the same radius from the center. If one of them is displaced inwards, its angular velocity increases. But by being retarded by the tether, it looses angular momentum and this tends to drive it in towards the center even more, so the
process can run away: there is an instability.

In visco-elastic polymer fluids, it has been known for some 20 years that there are similar types of instabilities due to the elasticity of the fluid. In this case, the polymers are stretched and oriented when there is a shear in the fluid. As a result, you can think of a sheared polymer fluid as being full of oriented stretched rubber bands. The visco-elastic instabilities that are known to occur are due to the combination of this stretching effect and the curvature of the flowlines. In this case, when a fluid element in a Taylor-Couette cell (two co-axial rotating cylinders with in between the polymer fluid) is displaced inwards, the polymer gets more stretched do to the higher shear rate. Due to the curvature of the streamlines and the orientation of the polymers the net force on a fluid element which is pointing inwards, then increases. As a result, there can also be a runaway effect (instability).

These visco-elastic instabilities have been explored extensively both theoretically and experimentally in the last years, and it is now known that beyond the instability thresholds such flows give quite rapidly rise to turbulence. Because the viscosity of polymer solutions is often quite high, inertial effects that give rise to turbulence in normal fluids like water, are usually negligible (the Reynolds numbers are small). One therefore speaks of elastic instabilities and elastic turbulence, and in most theoretical studies of elastic instabilities, inertial effects have been ignored completely.

The intuitive analogy between magnetic field lines that act like elastic tethers or elastic polymers actually extends to the mathematical level. Because magnetic field lines are advected by a fluid flow, the dyadic $BB$ that can be made of the magnetic field $B$ obeys an equation very similar to the so-called UCM model constitutive equation\(^1\) for the polymer stress field $\sigma$. Motivated by this analogy and the fact that for the astrophysical magnetorotational instability inertial effects are important, the authors proceed to search for visco-elastic instabilities in Taylor-Couette cells in the heretofore unexplored region of finite Reynolds numbers. Interestingly, they do find that there are indeed new instabilities that are the immediate analog of the magnetorotational astrophysical instabilities. The eigenmodes that exhibit these instabilities are clearly different from those that have been analyzed before in the extreme viscous limit as well as in the inertial regime. Since the elastic properties of polymer solutions can easily be tuned by changing the polymer length, this opens up the possibility the astrophysical instabilities in the laboratory.

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\(^1\)UCM model stands for Upper Convected Maxwell model — this is a Maxwell model for a flowing polymer fluid. The UCM model is exact for elastic noninteracting dumbbells in flow; even though it is difficult to handle (it is a nonlinear tensor equation) it is one of the simplest models for polymer rheology. The "upper convected" stands for a particular choice of nonlinear derivative that is dictated by frame invariance. The full equation is given in the paper by Ogilvie and Potter.