Quenching quantum many-body systems: fundamental relations between work distribution and dephasing.

The Statistics of the Work Done on a Quantum Critical System by Quenching a Control Parameter

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and

On quenches in quantum many-body systems: the one-dimensional Bose-Hubbard model revisited

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Recommended with a Commentary by Anatoly Polkovnikov, Boston University

Until recently first principles condensed matter research has been mainly focused on understanding equilibrium phenomena in many-particle systems. Non-equilibrium phenomena were mostly approached either phenomenologically or using kinetic equation which is usually applicable to weakly interacting systems not far from equilibrium. Equilibrium phenomena are described by the partition function, which is typically postulated to have a Gibbs exponential form. Lately new experimental systems like cold atoms have emerged which have stimulated wide interest in understanding properties of systems, especially quantum, which were driven away from the equilibrium and not coupled to the external heat bath. There are very few statements one can make with certainty about such systems. For example, there is a general belief that if sufficiently complex such systems will relax to thermal equilibrium. However, many issues remain illusive. In particular, what are exactly the conditions for such relaxation, how does relaxation happen, what are the other possible asymptotic (steady) states, what is the role of (non)integrability, dimensionality, range of interactions etc.?

In the paper highlighted above, A. Silva established a very interesting general connection between the distribution of the work done to perform a quench (a sudden change in the Hamiltonian) in an arbitrary system and the Loschmidt echo. If the system is initially in the ground state then the work distribution characterizes energy fluctuations in the system after the quench (note that the final state of the system does not have to be equilibrium). At the same time the Loschmidt echo (or fidelity) of the system is the measure describing dephasing in the equilibrium state due to the quench. It is formally defined as $L(t) = |G(t)|^2$, where

$$G(t) = \langle \exp[iH(g_0)t] \exp[-iH(g_1)t] \rangle \,. \tag{1}$$

Here g_0 and g_1 are the couplings before and after the quench and the expectation value is taken with respect to the equilibrium state corresponding to the coupling g_0. The Loschmidt echo is often used to characterize quantum chaos (see e.g. Ref. [1]). More specifically Silva establishes the equality between $G(t)$ and the fourier transform of the work distribution $P(W)$:

$$G(t) = \int dW \exp[-iWt] P(W). \tag{2}$$

This equality between the two seemingly different quantities suggests that there are possible fundamental relations between chaos (which governs the Loschmidt echo) and thermalization (which requires very specific energy distribution after the quench consistent with the equilibrium ensemble). This equality is also very closely connected to the Jarzynski's nonequilibrium work relation [2].

In the other paper highlighted above paper G. Roux develops a new numerical method, which allows one to directly compute diagonal (stationary) elements of the density matrix in a arbitrary system after a quench without need to compute the whole density matrix. These diagonal elements of the density matrix determine energy, all its moments and other conserved quantities in the system after the quench. They also determine all long-time properties of the system after it relaxes to the steady state (see e.g. Ref. [3]). Using ideas similar to Silva's, G. Roux develops an algorithm for computing the fourier transform of the quantity $A(t) = \langle \psi(t) | \psi(0) \rangle$ - the overlap of the wave function after the quench and the initial wave function. Up to an inessential phase, $A(t)$ coincides with $G(t)$ for the case of the pure initial state. In particular, $|A(t)|^2$ precisely gives the Loscshimdt echo $L(t)$ in the system. In turn the fourier transform of $A(t)$ contains all the information about the diagonal (stationary) elements of the density matrix. This method allows one to do calculations in much bigger systems then using the exact diagonalization and it also gives direct access to the long-time properties of the system (overcoming the main limitation of DMRG methods [4]).

The downside of the suggested method of course is that one does not obtain in this way the full information about dynamics in the system. The author illustrates his ideas with calculations to the quench in the Bose–Hubbard model and obtains very interesting results. In particular, he gets a strong indication that for small quenches the steady state density matrix in this model acquires canonical Gibbs form.

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