"A 'Granocentric' Model for Random Packing of Jammed Emulsions"

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Packing problems, such as how densely nonoverlapping particles fill d-dimensional Euclidean space, are ancient and still provide fascinating challenges for scientists and mathematicians [1,2]. Bernal has remarked that "heaps" (particle packings) were the first things that were ever measured in the form of basketfuls of grain for the purpose of trading or of collection of taxes. Understanding the nature of dense particle packings is a subject of intense research in the physical, mathematical and biological sciences.

Dense packings of hard particles have served as useful models to understand the structure of low temperature phases of matter, granular media, heterogeneous materials, and biological media. Finding the maximal-density packing arrangements is directly relevant to understanding the structure and properties of crystalline equilibrium phases of particle systems as well as their (zero-temperature) ground-state structures in low dimensions in which the interactions are characterized by steep repulsions and short-ranged attractions. Disordered jammed sphere packings have been employed to understand glassy states of matter [3]. There has been a resurgence of interest in maximally dense sphere packings in high-dimensional Euclidean spaces. Interestingly, the optimal ways of sending digital signals over noisy channels correspond to the densest sphere packings in high-dimensional spaces [2].

The preponderance of previous investigations have focused on dense packings of spheres in various dimensions. For congruent particles in three dimensions, the sphere is the only non-tiling particle for which the densest packing arrangements can be proved [4]. It is only very recently that attention has turned to understanding dense packings of congruent non-spherical particles in three-dimensional Euclidean space, including ellipsoids [5], superballs [6], and polyhedra [7,8].

It is a statement of the difficulty of the problem that we know very little about dense packings of spheres with a size distribution, despite the expenditure of considerable effort to study such systems, especially binary packings of spheres. Clusel *et al.* have carried out a beautiful series of experiments to understand polydisperse random packings of spheres. Specifically, they produce three-dimensional random packings of frictionless emulsion droplets with a high degree of size polydispersity, and visualize and characterize them using confocal microscopy. They also devised a simple statistical model in which the complexity of the global packing is distilled into a local stochastic process. They call this a "granocentric" model, which from the perspective of a single particle, the packing problem is reduced to the random formation of nearest neighbors, followed by a choice of contacts among them. The two key parameters in the model are the available space around a particle and the ratio of contacts to neighbors, and are directly obtained from experiments. They show that this "granocentric" view captures the local properties of the polydisperse emulsion packing, ranging from the microscopic distributions of nearest neighbors and contacts to local density fluctuations, and are also able to accurately predict the global packing density.

This work raises many interesting issues and questions for theorists. We are very far from a complete theory of random sphere packings. A local analysis, especially one that neglects even local correlations between particles, such as the granocentric model, cannot be expected to be fully predictive. But what type of modifications must be made to make a local analysis to improve its predictive capability? Any successful theory must be able to predict non-local properties, such as the pair correlation function for all particle pair separations, for example. It is known that disordered jammed monodisperse packings are characterized by a pair correlation function that exhibits a slow power-law tail decay (i.e., quasi long-range order) and have infinite-wavelength density fluctuations that vanish (i.e., they are "hyperuniform") [9]. Do these non-local characteristics persist in disordered polydisperse packings? It would be surprising if such packings prepared by standard protocols were not hyperuniform, but this question and the nature of the tail of the pair correlation could be probed using either computer simulations or confocal microscopy in experimental packings, as Clusel *et. al.* have already done in their interesting work. Although we know that disordered jammed monodisperse packings possess unusually slow decaying pair correlations, we currently have no fundamental explanation for this anomalous non-local attribute. Again, there are many challenges that lay before us that must be overcome to make progress on a comprehensive theory of random sphere packings.

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