

The Higgs mode in condensed matter.

M. Endres *et al.*, *The Higgs amplitude mode at the two-dimensional superfluid/Mott insulator transition*, Nature **487**, 454 (2012) ;

L. Pollet and N. Prokof'ev, *Higgs Mode in a Two-Dimensional Superfluid*, Phys. Rev. Lett. **109**, 010401 (2012).

The recent discovery of a particle with the characteristics of the Higgs boson of the Standard Model at the Large Hadron Collider has justifiably generated much excitement among physicists. Coincidentally, this was just preceded by observations of Higgs-like excitations in ‘table-top’ experiments on an ultracold atom system near the superfluid-insulator quantum phase transition.

For this discussion, the ‘Higgs’ mode is defined as an excitation associated with fluctuations in the amplitude of a field, $\vec{\phi}$, in a state in which a global or gauge symmetry has been spontaneously broken. In the simplest context, consider a Lagrangian with the relativistic structure

$$\mathcal{L} = \frac{1}{2} \left(\frac{\partial \vec{\phi}}{\partial t} \right)^2 - \frac{c^2}{2} (\nabla_x \vec{\phi})^2 - V(|\vec{\phi}|) \quad (1)$$

in a situation where the potential $V(|\vec{\phi}|)$ has a minimum at $|\vec{\phi}| = \phi_0 \neq 0$. Then an analysis of harmonic fluctuations about this minimum shows that in addition to the gapless Goldstone modes at frequency $\omega = ck$ (k is the wavevector), there is a Higgs mode corresponding to fluctuations in $|\vec{\phi}|$ at a frequency $\omega = \sqrt{\omega_0^2 + c^2 k^2}$ where $\omega_0 = V''(\phi_0)$. An important observation is that the relativistic structure of \mathcal{L} is crucial to the existence of the Higgs mode. If instead, we had performed the same analysis on the non-relativistic Gross-Pitaevski Lagrangian appropriate to the superfluid phase of a Bose gas

$$\mathcal{L}_{GP} = i\psi^* \frac{\partial \psi}{\partial t} - \frac{c^2}{2} |\nabla_x \psi|^2 - V(|\psi|), \quad (2)$$

then the amplitude and phase fluctuations of ψ become canonically conjugate to each other, and there is only a single Goldstone mode, but no independent Higgs mode.

In the particle physics applications, the symmetry of rotations of $\vec{\phi}$ is gauged by a Yang-Mills gauge field. We can perform a similar harmonic analysis of fluctuations close to $|\vec{\phi}| = \phi_0$: the Goldstone mode no longer exists and is instead absorbed into the longitudinal

component of a massive gauge boson. However, the Higgs mode survives, and remains similar to the excitation in the absence of the Yang-Mills gauge field.

In condensed matter, the low energy spectrum is generally not relativistic. However, in superconductors with the pairing energy $\Delta \ll E_F$, the Fermi energy, the effective action for fluctuations in the amplitude of the superconducting order, within the ordered superconducting phase only, has a relativistic form and so a Higgs mode is present (P. B. Littlewood and C. M. Varma, Phys. Rev. B **26**, 4883 (1982)). The recent condensed matter realizations of the Higgs mode are associated with quantum phase transitions which are fully described by relativistic Lagrangians like those in (1) in both the ordered and disordered phases, and at the quantum critical point between them. Ruegg *et al.* (Phys. Rev. Lett. **100**, 205701 (2008)) performed neutron scattering measurements on the three-dimensional insulating antiferromagnet TlCuCl_3 . Under ambient pressure, the $S = 1/2$ Cu ions are dimerized into nearly independent spin-singlet pairs. The quantum phase transition involves the onset of long-range antiferromagnetic order at zero temperature, and is induced by the application of pressure. The order parameter $\vec{\phi}$ has 3 components, and measures the antiferromagnetic correlations. Ruegg *et al.* observed an excitation with the characteristics of the Higgs mode in the state with long-range antiferromagnetic order: its energy, ω_0 , vanished upon approaching the quantum critical point in a manner which agreed quantitatively with the theoretical predictions of (1).

The experiments by Endres *et al.* are on the superfluid-insulator transition of ultracold atoms in a two-dimensional optical lattice; this quantum phase transition is described by the Lagrangian (1), with $\vec{\phi}$ now the 2-component superfluid order parameter. The low spatial dimensionality introduces a new subtlety not found in any of the other experiments. It is possible for the Higgs excitation to decay into multiple Goldstone modes, and the amplitude for such processes is infrared singular in two dimensions (note that this issue does not arise when the symmetry is gauged: the Goldstone boson is then gapped, and there are no physical infrared singularities). The analog of the neutron scattering experiment of Ruegg *et al.*, which measures the longitudinal spectral density of $\vec{\phi}$, would find it difficult to identify the contribution of the Higgs mode in two dimensions. Nevertheless, Podolsky and collaborators (Phys. Rev. B **84**, 174522 (2011); **86**, 054508 (2012)) have argued that signatures of the mode should be identifiable in probes which measure the spectral density of the modulus squared, $|\vec{\phi}|^2$; their second paper showed that the Higgs mode is associated

with a lower-half-frequency-plane pole in the response functions of both $\vec{\phi}$ and $|\vec{\phi}|^2$, whose real and imaginary parts vanish with the same exponent upon approaching the quantum critical point.

Endres *et al.* measured the response to a periodic modulation in the depth of the optical lattice across the superfluid-insulator transition. On the insulating side, they observed the expected gapped response of the Mott insulator, with the gap vanishing upon the approach to the superfluid. However, a finite frequency peak re-emerges on the superfluid side with a “pseudo-gap” at low frequencies: the frequency and lattice-depth dependence of this response are all consistent with the Higgs pole, along with its scaling properties upon approaching the quantum critical point.

Pollet and Prokof'ev carried out imaginary time Monte Carlo simulations on the Bose-Hubbard model across the quantum critical point. Their high precision data enabled analytic continuation of the $|\vec{\phi}|^2$ response function to real frequencies, and their results agree with the above theoretical and experimental studies.