RVB, spin liquids, and topological order

Norbert Schuch, Didier Poilblanc, J. Ignacio Cirac, and David Perez-Garcia, "Resonating Valence Bond States in the PEPS Formalism,"
Phys. Rev. B **86**, 115108 (2012).

Recommedned and a Commentary by David DiVincenzo, RWTH Aachen

What kind of ground state can a collection of quantum particles have, if that state exhibits no order? That question, pressed upon us first by the fact of liquidity of condensed helium at zero temperature, continues to have new answers in current work in quantum information theory. In particular, the work of Schuch *et al.* gives us new insights into the venerable Resonating Valence Bond (RVB) paradigm.

What RVB refers to as a "bond" is just the occurrence of singlet (zero angular momentum) pairing between two electron spins, not the full meaning of the bond in chemistry. The RVB idea is originally Pauling's. In his effort to understand why benzene does not dimerize, that is, why the π electrons do not choose particular partners and lower the symmetry of the ring to three-fold, he invoked the quantum-mechanical idea of resonance – degenerate pairing patterns (just two for benzene) in quantum superposition. Anderson extended this reasoning to solid state physics, to explain situations where symmetry breaking may not occur. His famous conjecture that the triangular antiferromagnetic Heisenberg lattice would fail to order, and would be described by an RVB state superposing a macroscopic number of different dimer configurations, was just barely wrong. But the fact that the three-sublattice Neel state slightly outcompetes the RVB spin liquid does not preclude RVB from being the right answer in a slightly different setting.

In any case, Anderson's work made RVB available as a paradigm for disordered quantum states, leading to concerted efforts, for example, to employ it for understanding the cuprates. In the original setting of Heisenberg magnets, it was understood that the triangular AFM was somehow not quite frustrated enough; we now are essentially certain that the Heisenberg AFM on other frustrated 2D lattices, for example Kagome, indeed has an RVB spin-liquid ground state.

Schuch *et al.* make this essentially airtight in the current work, by exhibiting a Hamiltonian on the Kagome lattice for which the short range RVB state (dimers only connecting nearest neighbors) is proved to be the ground state, and to have other interesting properties. The proofs are almost mathematically rigorous, some parts requiring the assistance of some extremely compelling numerics. This paper provides an intimate linkage between the RVB paradigm and other recently introduced paradigms for topological quantum ordering, including that coming from error correction code theory provided by Kitaev and others. But this paper shatters the necessity of a connection between frustration and RVB: its Hamiltonian is in fact by construction "frustration free", the RVB ground state is the lowest energy state of every term of the Schuch *et al.* Hamiltonian!

The PEPS (Projected Entangled Pair State) methodology that is used here looks specialized, but has rather transparent defining principles and has constructive and easily checkable consequences. The starting principle is that states that are candidates for quantum spin liquids on the one hand, or for the realization of topological order on the other, should obey the *area law*, which says that the entanglement between two regions of the lattice should be confined to near the boundary between the regions. The short range RVB itself obviously obeys the area law, but the PEPS construction is a more all-encompassing ansatz from which RVB and many other states can be produced. The projector scheme is powerful enough to give, from the same starting point, three very different looking models, with their associated states; first, the Kitaev toric code model; second, the dimer model of Rokhsar and Kivelson; third, a new model that has the short range RVB state as a ground state by construction. In fact it gives something even more powerful: a family of models that interpolate smoothly from the first to the second to the third model.

The last of these interpolations is a real curiosity. The dimer model is one in which, by fiat, different dimer coverings of the lattice are orthogonal. Part of the complexity of the real RVB model is that different dimer coverings have a finite inner product. Therefore, what is found is a family of models for which the degree of orthogonality of different coverings is varied continuously. The PEPS methodology makes available this interesting interpolation of states, and the associated projectors that force these states to be the ground states.

PEPS also permits easy numerical calculations of correlation functions, and other quantities, from which it is quite evident that there is no phase transition along this interpolation. Thus, the RVB ground space inherits all the interesting combinatorial features of the toric code: the ground space degeneracy is topological, being one on the sphere and four on a torus, independent of lattice size; all states in the ground state manifold are indistinguishable locally, and incapable of being transformed into one another by local means; and the excitiations above the gap are abelian anyons (but the numerics are not capable of giving direct evidence that the final RVB model is gapped).

A few final thoughts about the local Hamiltonian that gives this RVB state. Such projector constructions have a history going back at least as far as Affleck, Kennedy, Lieb, and Tasaki, but perhaps no previous ones have achieved the complexity of Schuch et al.; their projectors are rather extended structures involving whole patches of the lattice – involving 19 spins each. But as I said above, they formally are completely unfrustrated. The projectors act not by frustrating, but by declaring a lot of non-frustrated local arrangements to be illegal. For me, this looks a lot like a quantum verson of what happens with the matching rules in Penrose tilings: by forbidding enough local arrangements that lead to simple ordering, *all* ordering is forbidden, and something new (in Penrose's case, quasiperiodicity) must emerge.