Fractionalized two-dimensional states on surfaces of three dimensional topological insulators

1. A Time-Reversal Invariant Topological Phase at the Surface of a 3D Topological Insulator, Parsa Bonderson, Chetan Nayak, Xiao-Liang Qi, J. Stat. Mech. (2013) P09016

2. 4. Gapped Symmetry Preserving Surface-State for the Electron Topological Insulator Chong Wang, Andrew C. Potter, T. Senthil, arXiv:1306.3223

3. Symmetry Enforced Non-Abelian Topological Order at the Surface of a Topological Insulator Xie Chen, Lukasz Fidkowski, Ashvin Vishwanath, arXiv: 1306.3250

4. A symmetry-respecting topologically-ordered surface phase of 3d electron topological insulators Max A. Metlitski, C. L. Kane, Matthew P. A. Fisher arXiv: 1306.3286

Recommended with commentary by Ady Stern (Weizmann Institute)

The gedanken lab of a theorist working on topological states of matter is full, among other things, of lattices, flux tubes, and a box of wild cards called “symmetries”. The papers that I introduce here make use of these tools to further our understanding of surfaces of three dimensional topological insulators in situations where electron-electron interaction becomes important while time reversal symmetry and charge conservation hold.

In the absence of electron-electron interactions, these symmetries give rise to two classes of insulators, trivial and topological. Topological insulators carry gapless surfaces whose spectrum is characterized by an odd number of Dirac cones. When time reversal invariance is broken, the cone is gapped, and the surface turns into an integer quantum Hall state of a Hall conductivity of one half (in units of e^2/h). This is not an oxymoron. With each surface carrying a Hall conductivity of one half, the entire sample carries an integer Hall conductivity, thus justifying the name.

As we learned from the fractional quantum Hall effect, interactions may have a profound effect on topological states of matter. For three dimensional topological insulators two different types of effects should be distinguished. The first is the formation of a fractionalized three dimensional topological insulator, where fractional excitations exist in the three dimensional bulk. A transition from such a state to a non-interacting topological insulator requires a closure of the bulk energy gap. The second is the formation of a state that is essentially identical to a non-interacting topological insulator in the 3D bulk, but has a gapped surface even when time reversal symmetry and charge conservation are satisfied. It is this type of state that is the subject of the four papers that are "journal-clubbed" here.

Let us imagine a thickened torus made of a topological insulator with interaction between electrons being turned on only close to the surfaces, such that the bulk is in a non-interacting topological insulator state. In a gedanken experiment we use two flux tubes of half a flux quantum each (a flux quantum, known affectionately as a 2π flux, is hc/e). These flux tubes are turned-on, turned-off and time-reversed within the two holes of the torus, and the experiment leads to the conclusion that time reversal symmetry requires the surfaces of the torus to have at least one excitation with an excitation energy that approaches zero in the limit of an infinite torus. When there is a Dirac cone on the surface, it supplies a multitude of such states, and the requirement is more than satisfied. A gapped surface of the torus with several degenerate ground states satisfies the requirement as well, provided that the degeneracy "stable", i.e., cannot be lifted by local changes in the parameters of the systems. But that – a stable ground state degeneracy on a torus – is exactly the defining property of a fractionalized topological state, or a state with topological order. Thus, it is possible in principle to have a gapped surface that is time reversal symmetric and charge conserving, provided that it forms a fractionalized two dimensional topological state of matter.

And in practice? Well, let us first define here "in practice" as meaning "in the gedanken lab". The distance between the fractionalized surface states and the experimentalist's lab may still be a few journal clubs away. Under this definition, the four papers give a positive answer, and in an interesting way. With the goal being a gapped surface state that is time reversal symmetric and charge conserving, one possible route is to start with a gapless Dirac cone and gap it with a super-conductor. This achieves the desired gap, but at the expense of charge conservation. To reinstate charge conservation, a super-conductor to insulator transition needs to be induced. This transition is to be induced by the condensation of vortices within the 2D super-conductor. Condensation of vortices has been used as a way to describe a transition from a super-conducting state to an insulating state in several systems, e.g., an array of Josephson junctions. These systems are described in terms of two fields that are mutually conjugate – the density field and the phase field. A super-conductor is a state where the phase field is well defined, the density fluctuates strongly, and charge is not conserved. An insulator is a state where the charge density is well defined (think of a Mott insulator), the phase field fluctuates strongly and charge is conserved. The fluctuations of the phase may be described in terms of a fluid made by vortices and anti-vortices in equal numbers. When this fluid condenses to a superfluid, the phase field is completely disordered, charge is localized, and the array is an insulator.

This description of a superfluid to an insulator transition is an idealized one, leaving out all degrees of freedom other than the phase and the density. Within its idealized realm, it is based on the identification of vortices in a two dimensional super-conductor as quantum mechanical bosonic degrees of freedom, since only bosons Bose-condense.

The application of this description to the surface of a topological insulator is subtle, and there are two main reasons for that. First, a single vortex in that super-conductor is not a boson. In fact, the composition of the superconductor out of a single Dirac cone with no spin degeneracy leads to the formation of a localized zero energy Majorana fermion at the core of each vortex. The Majorana fermion makes the vortex into a non-abelian particle, rather than the bosonic vortex in a conventional two-dimensional super-conductor. And second, the vortex in the surface super-conductor carries also an electric charge, a quarter of the electron charge. To understand that, let us imagine a solid three dimensional annulus, whose upper surface is covered by the super-conductor, and the rest of the surface is in the quantum Hall $\sigma_{xy}=1/2$ state. A vortex is then inserted into the hole of the annulus by threading the hole with a flux tube of half a flux quantum. Due to the Hall conductivity of the surrounding region, the insertion of the flux is accompanied with an accumulation of opposite charges at the core of the vortex and at the interface between the superconducting and quantum Hall regions.

Altogether, single vortices cannot Bose-condense, and one needs to construct a group of vortices (four seems to be the smallest number possible) to create a bosonic particle. When these groups condense, the resulting state is insulating, gapped and fractionalized. With the condensation limited to groups of vortices, single vortices, or any other sub-group of the condensed group, remain as excitations that carry a fractional charge. Furthermore, due to the Majorana fermions in the cores of single vortices, their statistics is non-abelian. In fact, these excitations have some similarity with those of the Moore-Read quantum Hall state, although the state they are part of is very different. Not only that it is symmetric to time reversal, but it is a two dimensional state that can be realized only on the surface of a three dimensional topological insulator.

The gapped state that results from the process described in the four papers leads to a very interesting state, and raises many questions. For example, this is a gapped state that is symmetric to time reversal, and thus has a quantized vanishing Hall conductance. Since this is the surface of a topological insulator, a natural question is whether the application of an infinitesimal time reversal breaking term leads to a quantized Hall conductance of one half? Can an infinitesimal field close the energy gap, as required for a transition of a 2D system between two values of the Hall conductivity? The way the state is constructed guarantees that there are no gapless charge modes. Is the state fully gapped, or may there be gapless modes which do not carry charge? And perhaps the most interesting question, can the gapped state realize any type of non-abelian statistics, or only Majorana based ones?

I now leave the stage to four papers which could easily be described as well written, thought- provoking and original, but which I have a hard time describing as easy…