Dynamics of currents carried by the proximity effect in SNS-systems

- *Dissipation and Supercurrent Fluctuations in a Diffusive Normal-Metal-Superconductor Ring* B. Dassonneville, M. Ferrier, S. Guéron, and H. Bouchiat, Phys.Rev. Letters 110, 217001(2013)
- *Phase dependent Andreev spectrum in a difusive SNS junction. Static and dynamic current response* M. Ferrier, B. Dassonneville,S. Guéron, and H. Bouchiat, arXiv 1307.7961v2

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Introduction:
The 'superconducting proximity-effect' is a well-known set of words, which often is accompanied by a qualitative explanation such as *the leakage of Cooper pairs over a length called the normal metal coherence length* $\sqrt{\hbar D/k_B T}$. One step further it is reduced to the statement that a *normal metal takes on the properties of a superconductor and can be characterized by an induced gap* Δ_{ind}, from which point on the standard BCS expressions are used with a reduced gap.

In reality the superconducting proximity-effect is a very rich and complicated phenomenon of which many aspects are yet to be unraveled. The complexity arises because the actual superconducting properties are due to phase-coherent electronic states induced by the macroscopic quantum phase of the neigbouring superconductor, which in many cases are dependent on the actual geometry of the N-part and when two superconductors are present on the differences between their phases. In addition, analogous to the uniform BCS superconductor itself, the properties depend on the occupation of the states, which in equilibrium is the Fermi-Dirac distribution. In many experimental situations the system is driven and a non-equilibrium distribution will control the observable properties.

A great deal of the groundwork for the nonequilibrium superconducting proximity-effect was laid at the end of the 1960's up to the beginning of the 1980's, before the abundant availability of nano-structures. The latter started to become prominent for normal transport in the mid-1980's and continue to dominate until the present day. An important conceptual framework for *normal* transport in nanostructures is the Landauer-Büttiker formalism for quantum transport. It makes a distinction between a 'conductor' characterized by a transmission matrix and 'contacts', which are considered 'boring' equilibrium reservoirs. They serve as equilibrium baths for emitted waves and also serve as reservoirs in which thermal equilibration of transmitted and absorbed waves occurs. Such a model has served very well for GaAs/AlGaAs heterostructures, where the conductor and the contacts are made of one and the same material. Similarly with mechanical break junctions. In the latter case the framework can also be applied to study in great detail the equilibrium and nonequilibrium superconducting properties of, for example, single-atom point contacts.

The situation is much more complicated when the conductor and the contacts consist of dissimilar materials. In many cases some kind of nano-conductor, nanotube, nanowire, molecule, 2DEG, etc is contacted to one or two superconductors. Experimentally one finds, obviously, the Josephson-effect. Any kind of coupling of the two macroscopic quantum phases of the two superconductors will produce a supercurrent, which depends periodically on the phase-difference, will produce Shapiro steps or some form of the Fraunhofer-diffraction pattern (which is geometry dependent). Being a macroscopic quantum-effect these properties are fairly universal and therefore do hardly reflect specifics of the objects under study.

The interesting and urgent questions for the hybrid structures, in my view, are what are the phase-coherent microscopic states that carry the supercurrent, for zero voltage and at finite voltage, and how these are populated in the static and in the dynamic case. Although, one would wish that the conceptual framework can be as easily digestible as the Landauer-Büttiker framework the reality is most likely that the noequilibrium proximity-effect theory, known as the quasiclassical theory for nonequilibrium superconductivity will play a key role. And of course for the more ballistic case a tractable version of the Bogoliubov-De Gennes equations. The theory will have to be extended and applied to treat new experimental conditions. For example, although boundary conditions for a system between a diffusive superconductor and a diffusive normal metal as well as for two coupled ballistic systems are known, the boundary conditions for an interface between a ballistic normal metal system and a diffusive superconductor are not known. In fact, they may require detailed knowledge of the interface, often not known in experiments.

Dynamic effects in diffusive superconducting constrictions:
An example of properties studied is voltage-carrying state of superconducting constrictions such as pointcontacts and microbridges. Unfortunately, most of this type work is missing in the often used review of Likharev[1]. The review does not include the progress that was made in nonequilibrium superconductivity, in which the energy-dependence of many quantities plays a role and which was at the time of writing, 1978, still under development, in particular in its application to superconducting weak links.

In superconducting constrictions the current density in the constriction is the highest. Therefore the system behaves in many ways analogous to a SNS junction with a short N-channel, in which the weakening by the transport current is counter-acted by the presence of the nearby equilibrium superconductors. One of the earliest expressions for the DC supercurrent has been given by Aslamazov and Larkin[2].

For the voltage-carrying state the elementary first attempt to understand the current-voltage characteristic was based on the engineering-model introduced by Stewart and McCumber, the RSJ-model. It assumes a Josephson-element characterized by the two Josephson-equations $I_s = I_c \sin\phi$ and $d\phi/dt = 2eV/\hbar$ in parallel with a normal conductor with a current given by $I_n = V/R$. For a current-driven system one finds the well-known hyperbolic shape of the current-voltage characteristic for the time-averaged voltage: $< V >= 1/R\sqrt{I^2 - I_c^2}$. This model makes it easy to understand that the observed I,V curve is a time-average. In addition, one can use this model also to calculate the response to an external microwave field to generate the so-called Shapiro-steps.

This engineering-model is very helpful from a pedagogical perspective to understand the rough features of constriction-type superconducting junctions, as well as many others. However, there are no real systems which actually show the behavior contained in the RSJ-model. The only exception is a real tunnel-junction on purpose shorted by a normal metal strip, as often used in SQUID-devices. For unshorted, genuine Josephson-junctions the RSJ-model ignores a variety of properties, which originate in the microscopy of the energy-dependent properties of the coherent states in the constriction. Therefore they are all different in the details. To illustrate this point I take the symbols used in the RSJ-model. In contrast to what is assumed in the RSJ-model, the amplitude of the supercurrent I_c is *not* voltage-independent, the normal conduction is also *not* voltage independent, and the dependence on $\sin\phi$ is often *not* correct. Finally, there is also the so-called $\cos\phi$-term which is a quasi-particle pair interference term, also omitted in the RSJ-model.

Here, in the present Commentary, triggered by the 2 papers of Ferrier, Dassonville, Gueron and Bouchiat, the focus is only on the voltage dependence of the supercurrent $I_c(V)$. For superconducting constrictions the value of $I_c(V)$ at finite voltages can and, in fact in many cases, is different from the one at $V = 0$. The latter can easily be determined, but the former at finite voltages is hidden in the time-averaged value. It is found experimentally that with increasing voltage the amplitude rises proportionally with the dc voltage until it reaches a voltage at which the Josephson-frequency, the frequency of the oscillating voltage, equals the inelastic scattering time of the superconducting material used[5]. At that point the amplitude increase saturates and stays fixed for increasing current. This phenomenon is called 'dynamic enhancement' of the critical current[6].

This behavior is due to the following physical process[3, 4, 7]. For simplicity of the argument we assume a dc voltage-bias. In the Josephson-effect the distribution of current-carrying states in the constriction is energy-dependent and the actual dependence is on its turn controlled by the phase-difference (see for example Baselmans et al[8]). For $V = 0$ with increasing current the phase-difference adjusts itself to be compatible with this current. At finite voltage the Josephson-relation, $d\phi/dt = 2eV/\hbar$, leads to a linear increase of the phase difference, which microscopically means that the current-carrying density of states oscillates in time. To mimic this pattern Tinkham[6] used the terminology of a relaxation oscillation. For a voltage the supercurrent is accelerated and with increasing supercurrent the critical current is reached, where the superconducting state collapses and the constriction is temporarily quasi-normal. At this point in time the current is carried fully as a normal current and the superconducting state can rebuild, starting to carry also a supercurrent. This relaxation oscillation occurs at the Josephson-frequency.

The crucial understanding is that with increasing voltage, i.e. increasing Josephson frequency, the density of states in the constriction is changing so rapidly that it gets populated with some delay, making it dissipative, and the average supercurrent is affected in the sense that it increases in amplitude. The important parameter is the energy-relaxation time τ_{in}, which controls the tendency to equilibrium by electron-phonon processes. For $eV << \tau_{in}$ the supercurrent is out of phase by $\pi/2$ and therefore appears dissipative. For $eV >> \tau_{in}$ the supercurrent is in phase with the static part. This pattern of enhanced supercurrent for finite voltages is called dynamic enhancement in analogy to the enhancement of superconductivity in films in a microwavefield as first predicted by Eliashberg.

Dynamic effects in diffusive SNS systems:
In the recent articles by Ferrier, Dassonville, Gueron and Bouchiat, the same theme is being addressed but now in modern well-defined nanostructured SNS systems. Although the proximity-effect in NS was introduced in the early 60's, shortly after Josephson's prediction, their quantitative research started much later. The reason was that their impedance, before the advent of nanostructuring, was much too low for practical applications. Therefore there was much more interest in tunnel-junctions as well as in constricton-type junctions in contrast to the very low-impedance SNS junctions.

Theoretically the one-dimensional ballistic case for SNS junctions was addressed early on by Kulik, Ishii and Bardeen & Johnson, as due to one-dimensional Andreev bound states. Such ballistic systems have only recently become experimentally accessible. The diffusive case has become accessible by the gradual progress in nanofabrication, initially by combining semiconducting 2DEGs with superconducting contacts and later with ordinary metals combined with superconductors. In principle the theory to address the diffusive case, the quasiclassical theory for diffusive systems, was already available, but controlled experimental data were needed to provide mutually stimulating progress.

In a diffusive SNS system the density of states in N is dependent on the phase-difference and shows a so-called mini-gap, which is related to the Thouless energy. The current carried by these states is controlled by their occupation numbers. This was very elegantly demonstrated by Baselmans et al.[9] in which by tuning the occupation numbers they were able to create a π-junction i.e. altered the fundamental current-phase relation showing that $\sin\phi$ is not a fundamental relationship of a SNS Josephson-junction but one that results from an appropriate integral over the energies, for a given distribution-function.

In the experiments reported by Ferrier, Dassonville, Gueron and Bouchiat they take a well-defined diffusive SNS system. They use two different conceptual schemes to discuss the results. On the one hand they speak about a dense Andreev spectrum in N, on the other hand they apply the quasiclassical nonequilibrium theory (which they call Keldysh-Usadel). The essence is the same in the sense that they envision an energy and phase-difference dependent density of states in N. The SNS devices is part of a superconducting loop, which allows the tuning of the macroscopic phase-difference with a magnetic field (a well-known technique often called Andreev-interferometry). The SNS device is also coupled to a multi-mode superconducting resonator. It allows them to measure the resonant response of the resonator for changes in the impedance of the SNS loop. The conditions of the loop are changed by an applied flux, which is equivalent to applying a phase-difference across the SNS junction. While modulating the flux they monitor the change in eigen-frequency as well as the change in quality-factor of the resonator. Through this they obtain the complex susceptibility, which contains the information about the in-phase and out-of-phase component, or the dissipative and the kinetic inductance component of the currents carried by the SNS device.

The measurements are carried out over a frequency range of 190 MHz to 3 GHz and over a temperature range of 0.4 to 1.5 K. The frequency range is important because it allows them to cover a regime where τ_{in}^{-1} is smaller or larger than the frequency. The relaxation time τ_{in}^{-1} is assumed to be determined by a thin layer of Pd between the niobium and the normal metal. The not too low temperature range allows them to maintain a $k_B T >> E_{minigap}$. The experimental observations are quite clear in demonstrating the emergence of the dissipative component. They observe clearly the change from 2π to π periodicity with increasing frequency. The data appear to be consistent with what is to be expected qualitatively for a SNS junction and the two articles should further speak for themselves.

They supplement their analysis with calculations based on the Bogoliubov-DeGennes equations with on-site disorder using an Anderson-model. In addition they compare the results also with results based on the Keldysh-Usadel equations[11]. They find excellent agreement for the low frequency regime, but differences in the high frequency regime. Possible differences are interesting because the Usadel equations are for impurity-averaged Green's functions, whereas in some cases one is interested in the ballistic limit and the cross-over from diffusive to ballistic processes is very important for many of the current experiments.

The subject addressed in these articles revives also much older work by Lempitsky [10], which focused on predictions for the current-voltage charcateristic of a SNS junction, in analogy for those observed for constriction-type devices and addressed in Section 2 of this Commentary. In addition it touches upon the older debate of what causes the microwave-enhanced supercurrent in constriction-type microbridges, known as the Dayem-Wyatt effect. After it was established that the superconducting properties as such can be enhanced by microwaves, the analogous effect for a constriction-type microbridge was regularly discussed, most recently by Bergeret et al[12]

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