Waving transport and propulsion in a generalized Newtonian fluid
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and
Physics of rheologically enhanced propulsion: different strokes in generalized Stokes
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Recommended with a commentary by Thomas R. Powers, Brown University

The hydrodynamics of swimming microorganisms continues to be an area of active research. Although much of the theoretical framework for studying individual microscopic swimmers in water is well-developed [1], many questions about swimming in complex fluids such as mucus remain to be answered. Mucus is a non-Newtonian fluid—it is viscoelastic, with a fading memory, and its viscosity depends on shear rate (Fig. 1, left panel). The effect of viscoelasticity on swimming has been studied theoretically, computationally, and experimentally (see e.g. [2, 3, 4, 5, 6]). The shear-rate dependence of viscosity has received much less attention until now. It is important to understand these non-Newtonian effects, since swimmers ranging from mammalian sperm to infectious bacteria commonly encounter mucus. Furthermore, it has recently been estimated that shear-thinning effects should be more important than viscoelasticity [7]!
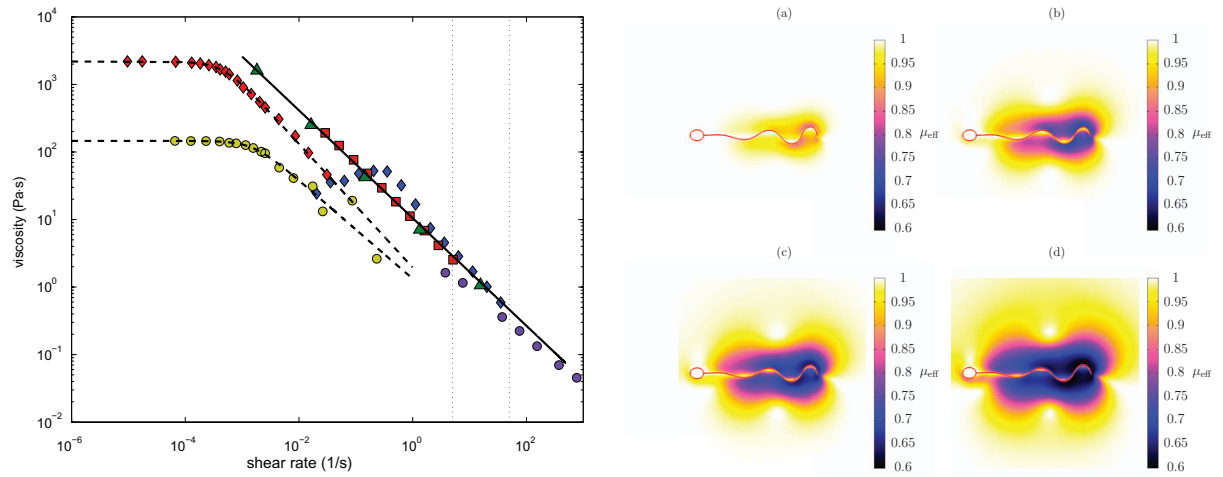

Figure 1: Left panel: Viscosity of various forms of mucus as a function of shear rate: human sputum (blue diamonds and red squares), pig intestine mucus (green triangles), human cervicovaginal mucus (purple and green circles), and human lung mucus (red diamonds). Figure adapted from [7]. Right panel: spatial dependence of viscosity on beat frequency for (a) $\omega\lambda = 0.2$, (b) $\omega\lambda = 0.8$, (c) $\omega\lambda = 1.5$, and (d) $\omega\lambda = 3$. Figure adapted from [8]

The Carreau model describes the shear-rate dependence of mucus:

$$\mu_{\text{eff}} = \mu_0 \left[1 + (\lambda|\dot{\gamma}|^2)\right]^{(n-1)/2},$$

1

where μ_0 is the viscosity when the shear rate $\dot{\gamma}$ vanishes, λ is a time constant that determines the shear rate at which the viscosity begins to depart from the zero-shear-rate viscosity, and n is the dimensionless power law index. When $n < 1$ the fluid is shear-thinning, and when $n > 1$ the fluid is shear-thickening.

In their paper [7], Vélez-Cordero and Lauga use the Carreau model to investigate the swimming and transport of the Taylor sheet, a workhorse model geometry for swimming microorganisms. The Taylor sheet is an infinite thin sheet subject to prescribed small-amplitude transverse or longitudinal deformations. Flagellum-propelled swimmers are typically modeled by an inextensible Taylor sheet. When modeling the coordinated beating of cilia coating the airway or the surface of a *Paramecium*, the extensible Taylor sheet is appropriate. The swimming speed or fluid transport is calculated by expanding in powers of the amplitude of the wave. Working to fourth order in amplitude, Vélez-Cordero and Lauga find that the swimming speed for an inextensible sheet is the same in a shear-thinning fluid as in a Newtonian fluid, although the swimmer is more efficient in a shear-thinning fluid. Interestingly, Montenegro-Johnson, Smith, and Loghin find an *increase* in swimming speed in shear-thinning fluids in their computational treatment of finite-length swimmers with large amplitude waves that increase with distance along the flagellum (Fig. 1, right panel). They attribute the increase in speed to a gradient in shear viscosity along the flagellum. As Vélez-Cordero and Lauga suggest, this mechanism could be further studied analytically by modifying the Taylor sheet to account for the increasing amplitude. Both these papers are important because without analytic calculations and numerical calculations in parallel with quantitative experiments, it is very difficult to isolate the physics governing microorganisms swimming in complex fluids.

References

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