

# Hydrodynamic long-time tails after a quantum quench

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Ever since statistical mechanics was formulated, the question of how a thermodynamic system in an initial non-equilibrium state approaches equilibrium has been of central interest. In the usual situation of a condensed matter system coupled to a heat bath and a particle reservoir the approach to equilibrium is governed by collision processes and transport processes inside the system and dissipation in the reservoirs. As a result, the equilibration occurs exponentially fast, the relaxation rates being determined by the typical energy scales in the system. With the advent of experiments on ultracold atomic gases one now has a new class of many-body systems inviting studies of the relaxation to equilibrium out of an initial non-equilibrium state. These systems are closed quantum systems, which means that there are no reservoirs available speeding up the relaxation process. Non-equilibrium initial states may be readily prepared by rapidly changing confining potentials and even the interparticle interaction (for a recent experimental study see [1]).

While theoretical studies of this fundamental problem in the time period before the development of experimental tools to study ultracold gases have been scarce, during the past decade a number of studies, both analytical and numerical, have been performed to understand how (or if) a quantum many body system relaxes to equilibrium after a "quantum quench" [2,3]. The general features of the equilibration process have been identified as a first short time adjustment of the initial state (usually the ground state of the undisturbed system) to the new Hamiltonian, as a second stage of "prethermalization" into a quasi-stationary intermediate state (involving the formation of the new quasi-

particles)[4], as a third stage where the scattering of quasiparticles establishes local equilibrium, and finally a last stage in which global equilibrium is reached by means of transport processes.

The last two steps are commonly thought to involve exponentially fast relaxation. As pointed out by Lux et al. this may not be true for isolated systems not coupled to reservoirs (which are by definition in thermodynamic equilibrium). As has been known since the 1970s, certain correlation functions of a closed system show so called long-time tails, characterized by a slow power law decay in time as opposed to the usual exponential decay. Lux et al concentrate on the energy density correlation function and show in the supplementary material how the long time tails may be derived within a hydrodynamic description incorporating fluctuations. They consider the bosonic Hubbard model in one dimension (far from an integrable point and without momentum conservation), which has two types of quasiparticles, holons and doublons. At not too strong interaction they use a semiclassical description allowing to simulate  $10^5$  quasiparticles and 500 different systems, over which the average is taken. The long-time tails are clearly visible in the energy-density correlation function, the average energies of doublons and holons, respectively, and other quantities. It is also shown that the set of correlation functions describing averages of the product of energies of the  $i$ -th and the  $(i+n)$ -th particle obeys scaling. As long-time tails were originally discovered for classical systems, it is also interesting to look at the strong quantum limit, which is the limit of strong boson interaction. In the absence of suitable analytical methods for that case the authors employ exact numerical diagonalization and show that long time tails emerge there as well, at least for a 14-site system at strong coupling.

The work of Lux et al. demonstrates that the interpretation of quantum quench experiments is more involved than previously thought. It would be highly desirable to observe the predicted long-time tails in experiment.

References:

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