

Non-Fermi Liquid Phases and Intertwined Orders in Semimetals with Quadratic Band-Touching

E-G. Moon, C. Xu, Y. B. Kim, and L. Balents, Non-Fermi liquid and topological states with strong spin-orbit coupling, *Phys. Rev. Lett.* **111**, 206401 (2013).

O. Vafek, J. M. Murry, and V. Cvetkovic, Superconductivity on the brink of spin-charge order in doped honeycomb bilayer, *Phys. Rev. Lett.* **112**, 147002 (2014) and M. Murray and O. Vafek, Excitonic and superconducting orders from repulsive interaction on the doped honeycomb bilayer, *Phys. Rev. B* **89**, 205119 (2014).

Recommended with a commentary by S. A. Kivelson

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The existence of a Fermi surface is the special feature of metals that distinguishes them from other quantum phases of matter, and is also the source of the formal difficulties in treating them when the usually successful Fermi liquid description breaks down. In particular, the nature of the interrelation between different broken symmetry phases and issues related to the existence of putative non-Fermi liquid phases are still being actively debated – the issue being that a quantum field theoretic solution is much more difficult to obtain for any state in which the low energy excitations are not concentrated at isolated points in k-space. In this context, the study of correlation effects in semimetals is a promising arena for studies of emergent properties of interacting fermions without the (fascinating) complexity of a Fermi surface. Needless to say, there has been a flurry of excitement in the past decade concerning the behavior of semimetals with a Dirac spectrum, inspired by work on graphene and topological insulators. However, from a formal viewpoint, band structures with a quadratic band-touchings are still more interesting, as they have a higher density of states at low energy, making interaction effects more important. Two recent papers have exploited this feature to obtain particularly illuminating results: A) On the basis of a renormalization group (RG) treatment using an ϵ -expansion about spatial dimension $d=4$, Moon *et al*[1] have shown (following on classic work by Abrikosov[3]) that in the presence of long-range Coulomb interactions there exists a stable and readily characterized non-Fermi liquid phase (which there is every reason to believe extends smoothly to $d=3$), in which there are gapless fermionic excitations but no well-defined quasiparticles. B) Vafek *et al* [2] have focused on $d=2$, where a large number of short-range interactions are marginally relevant[4, 5]; they have used a

perturbative RG approach to study the way in which a complex non-linear feedback between the various ordering tendencies involved leads to a situation in which, on intermediate length and temperature scales, these orders are intertwined. Under these conditions, small changes in the microscopic parameters can tilt the balance between a number of possible ordered phases.

One can think of a quadratic band touching point as arising from the coalescence of a pair of (more familiar) Dirac nodes. It is then easy to see that (at band-structure level) the perturbative stability of such a point must be protected by an appropriate point-group symmetry. (This was, presumably, well known in the storied past – for a recent pedagogic discussion of this point, see [4].) The Moon *et al* paper[1] considers as a potential physical realization of the problem certain iridates which are known to have the appropriate features in their band-structures. Vafeek *et al*[2] frame their discussion in the context of bilayer graphene, where the point-group symmetry is insufficient to produce a stable quadratic touching point, but where it is generally agreed that the band-structure effects associated with trigonal warping (which tend to split putative quadratic point into three Dirac and one anti-Dirac nodal points) are sufficiently weak that interaction effects are likely more important.

The vanishing density of states in $d > 2$ implies that short-range interactions are irrelevant at a quadratic band-touching. However, long-range (e^2/r^{d-2}) Coulomb interactions are perturbatively relevant in $d < 4$. What Moon *et al* have shown is that for $d = 4 - \epsilon$ with $0 < \epsilon \ll 1$, there is a nearby fixed point at which the effective coupling is small but non-zero, roughly $e^2 \sim \epsilon$, so the properties of the interacting fixed point can still be addressed perturbatively. This fixed point corresponds to a non-Fermi liquid phase with (calculable) power law correlations. For instance, the low-temperature specific heat $c_v \sim T^{d/z}$ where the dynamical exponent $z = 2 - \epsilon [4/(15N + 4)] + \mathcal{O}(\epsilon^2)$ where N is the number of flavors of electron. There is every reason to think that these results extend smoothly to $d=3$, which Moon *et al* further supported by analyzing the problem in the large N limit. Extrapolating these results to $d=3$ and $N=1$, this gives $z \approx 1.8$ and $c_v \sim T^{1.7}$. Note that, in contrast, the non-interacting system would have a larger low-temperature specific heat, $c_v \sim T^{1.5}$.

There are many cases in which conducting materials fail to exhibit Fermi liquid behavior. The idea that these anomalies might represent the existence of a non-Fermi liquid phase has always been attractive, although somewhat less so when such phases were defined by what they are not, rather than by what they are.

The structure of the problem is entirely different in $d = 2$ where there is a finite density of states at the non-interacting fixed point. As consequence of this, a variety of short-range interactions are marginally relevant[4, 5], and consequently there is runaway flow indicating tendencies toward

a variety of possible broken symmetry states.[2] Some of these are insulating (*e.g.* an anomalous quantum Hall phase which breaks time reversal symmetry and an anomalous quantum spin-Hall state which doesn't) and some are still semimetals (including various forms of charge and spin-density wave states which break translational symmetry, nematic and related states which break point group symmetries, and loop current states which break time-reversal symmetry while preserving spin rotational symmetry). For repulsive interactions, none of the leading instabilities is toward a superconducting state, but a small shift of the chemical potential from the band-touching point (which is a highly relevant perturbation) results in strong tendencies toward various forms of unconventional superconducting order and, in some circumstances, a pair-density wave phase – a spontaneous (and commensurate) version of an FFLO state. What is particularly notable about these results is that, as a consequence of the non-linearity of the RG flow equations, the interactions in the various channels are coupled to each other such that the growth of one sort of ordering tendency drives the growth of others. In this sense, the various orders are “intertwined.” The RG equations have a formal structure that is similar to that seen in old perturbative studies of the two-leg Hubbard ladder;[6] in that case the flows converge to a “ray” with an emergent higher symmetry ($SO(8)$ in this case) which would imply that various physically distinct energy scales in the problem are equal. It is unclear whether there is any similar emergent symmetry associated the flows in the present problem; none has been identified to date.

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