In 1950s, Bismuth single crystal was quantum theory of metals’ darling. Finding its Fermi surface geometry by means of various galvanomagnetic effects was a triumph of the “fermiology” [1]. The new era in the electronic band structure theory ushered by the notion of topological insulators [2, 3] may return Bismuth, now in the form of atomically-thin layers, to the center stage.

As explained in the Supplementary information of arXiv:1404.2598, theory predicts a helical electron band localized at the edge of a Bi bilayer. Due to the strong spin-orbit interaction, only one state per $k$ belongs to the edge mode ($k$ is a one-dimensional wave vector parallel to the edge). However, the energy spectrum $E(k)$ of the edge mode is peculiar. It is not monotonic, having a minimum $E_{\text{min}}$ at $k = 0$ and two side maxima $E_{\text{max}}$ at some $k = \pm k_0$. As the result, in the energy interval $E_{\text{max}} \geq E \geq E_{\text{min}}$ there are two Kramers pairs for every energy value $E$. According to Fig. S4(a), at sufficiently large $|k|$ spectrum $E(k)$ drops below $E_{\text{min}}$. At these energies ($E < E_{\text{min}}$) the edge mode carries only one Kramers doublet ($\pm k$) per energy level $E$. The coveted helical edge behavior, characterized by the absence of potential backscattering should develop in that very energy interval.

We note in passing that the Bi bilayer is not an insulator. The entire edge electron band exists on the background of the two-dimensional continuum. As arXiv:1404.2598 puts it, the edge states which live in a “momentum-energy window . . . coexist with the metallic surface states”.

Drozdov et al ventured to map out the edge electron band by means of STM measurements. STM can be used to infer the energy dependence of the electron density of states, $\nu(E)$, from the nonlinear tunneling $I - V$ characteristic. A more subtle use of STM sometimes allows one to extract the electron dispersion relation $\varepsilon(\vec{k})$ of various surface modes.
from the properties of standing waves. A standing wave is formed by backscattering from an imperfection; the spatial period of the standing wave depends on electron energy, thus yielding \( k(\varepsilon) \) for a given direction of \( \vec{k} \). The preprint applies both techniques to the edge states of Bi.

For now, only small islands of Bismuth bilayers on top of a cleaved surface of a Bi single crystal are available. The islands’ electron states are hybridized with the underlying bulk Bi, but by a strike of luck the electron states associated with edges stretching in certain directions (“type-A edges”) are hybridized only slightly. The typical state broadening is about 20% of its energy measured from the corresponding band edge. The tunneling \( I-V \) characteristics reveal a square-root (van Hove) singularity, \( \nu(E) \propto 1/\sqrt{E_{\text{max}}-E} \), in the edge density of states at \( E = E_{\text{max}} \). The singularity is consistent with the predicted parabolic maxima of \( E(k) \) at \( k = \pm k_0 \). Further progress is achieved by investigation of the electron standing waves. The ends of type-A edge segments may backscatter electrons thus leading to standing waves. The potential backscattering is allowed only in the energy interval \( E_{\text{max}} \geq E \geq E_{\text{min}} \) where scattering between the states belonging to two different Kramers pairs is possible. Experiment indeed reveals standing waves only in the said interval. The standing waves are seen best at energies \( E \) slightly below \( E_{\text{max}} \) where the Authors attribute them to the interference between states such as \( k = k_0 + q_1/2 \) and \( k = k_0 - q_1/2 \), which indeed belong to different Kramers pairs.

In summary, the current experimental results do provide spectroscopic information about the edge states in the energy interval \( E_{\text{max}} \geq E \geq E_{\text{min}} \) and are consistent with the conclusion that the edge state in a Bi bilayer is helical and has a non-monotonic dispersion \( E(k) \). However, the intriguing energy-momentum domain \( E(k) < E_{\text{min}} \), where one expects no potential backscattering, remains unaccessible. Possibly, designing an experiment with controllable spin-dependent scattering, thus allowing backscattering within a Kramers pair may transform STM into a versatile device for studying helical edges. In any case, STM emerges as an interesting alternative to local-probe methods used recently [4, 5] to demonstrate the presence of edge states in 2D topological insulators formed in semiconductor heterostructures.


