Wiring Up Topological Insulators

Time Reversal Invariant Topologically Insulating Circuits

Authors: Jia Ningyuan, Ariel Sommer, David Schuster, Jonathan Simon

arXiv:1309.0878

Topological properties of linear circuit lattices

Authors: Victor V. Albert, Leonid I. Glazman, Liang Jiang

arXiv:1410.1243

Spin-orbit-free Topological Insulators without Time-Reversal Symmetry

Authors: A. Alexandradinata, Chen Fang, Matthew J. Gilbert, B. Andrei Bernevig

Phys. Rev. Lett. **113**, 116403 (2014)

Recommended and Commentary by Steven M. Girvin, Yale University

Within the tight-binding approximation (which we assume here purely for simplicity), Bloch's theorem tells us that the solution of the (one-body) Schrödinger equation for a system with N bands has the simple form

$$\Psi_{\vec{k}}(\vec{r}) = \sum_j e^{i\vec{k}\cdot\vec{R}_j} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_N \end{pmatrix}_{\vec{k},j}, \tag{1}$$

where the sum is over all lattice points $\vec{R}_j$, and the column vector is shorthand for the function consisting of the coherent superposition of the atomic orbitals φ

$$u_{\vec{k},j}(\vec{r}) = \sum_{m=1}^{N} \alpha_m(\vec{k})\varphi_m(\vec{r} - \vec{R}_j). \tag{2}$$

The wave vector $\vec{k}$ lives in the first Brillouin zone, and because its boundary points are identified with points on the opposite boundary (since they are connected by a reciprocal lattice vector) the topology of the first Brillouin zone is that of a (hyper-)torus. Eq. (1) defines a continuous mapping from the torus to the manifold of N-dimensional complex vectors (of unit norm). The recognition that this mapping can be topologically non-trivial has been the source of considerable excitement in the field in recent years. One of the remarkable implications is that there exists more than one kind of band insulator. An insulator whose band structure has non-trivial topology can be insulating in the bulk but have gapless edge modes (for fundamental topological reasons independent of many details of the Hamiltonian). Various aspects of this novel physics has been observed in 2D and 3D materials, cold atomic gases, and in photonic materials [1-3].

In a novel experimental paper, Ningyuan et al. have created an artificial topological 'insulator' out of a periodic array of inductors and capacitors with cleverly arranged interconnections. This linear circuit is of course simply a collection of harmonic oscillators whose RF photon excitations are bosonic, but the band structure describing the collective mode dispersion is precisely equivalent to that of a (lattice) free-electron fermionic model. By exciting the array at a frequency inside the band gap of the bulk band structure, Ningyuan et al. are able to see the propagation of edge modes around the boundary of the circuit array.

Inspired by this initial paper, Albert et al. have carried out a general theoretical analysis of the topological features of linear electrical circuits. In topological insulators with time-reversal symmetry but strong spin-orbit coupling, the left- and right-moving edge modes are time-reverses of each other, with opposite wave vector and opposite spin (because electron spin is odd under time-reversal). If there is disorder is present, it cannot cause back scattering between the edge modes as long as the scattering is non-magnetic since it cannot flip the spin. In the case of electrical circuits, the interconnections are designed so that the amplitude on different sites within the unit cell defines the orientation of a pseudo-spin. While this pseudo-spin is odd under a certain anti-unitary symmetry associated with the fact that the equations of motion are real, it is *not* odd under conventional time-reversal. Hence the edge modes are protected only by the fine-tuning of the electrical component values which must be the same in each unit cell. This point is closely related to ideas of Alexandradinata et al. who have developed a theoretical analysis for condensed matter systems in which the boundary modes are protected by the point group symmetry of the bulk crystal rather than time-reversal.

The freedom to create 'designer' band structures and the ability to measure the phase and amplitude of the collective modes as they propagate in an electrical array will open up new vistas for direct 'wave function measurement' not obtainable in 'real' materials. In addition, it may be possible to engineer higher pseudo-spins and more complex symmetries than in ordinary materials. It will be interesting to see further experimental developments with electrical circuit arrays, perhaps including the addition of superconducting qubits to give interactions among the bosons in the presence of a band structure with non-trivial topology, another topic of growing interest in connection with cold bosonic atoms in optical lattices.

References:

1. 'Colloquium: Topological insulators,' M.Z. Hasan and C.L. Kane, *Rev. Mod. Phys.* **82**, 3045 (2010).

2. 'Topological insulators and superconductors,' Xiao-Liang Qi and Shou-Cheng Zhang, *Rev. Mod. Phys.* **83**, 1057 (2011).

3. 'Topological photonics,' Ling Lu, John D. Joannopoulos and M. Soljačić, *Nature Photonics* **8**, 821-829, (2014).